Quiz 2

MATH 2300-001

September 2, 2008

1. Find $F''(2)$ where $F(x) = \int_0^x f(t)\,dt$ and $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^2}}{u}\,du$.

$$F'(x) = \frac{d}{dx}\left[\int_0^x f(t)\,dt\right] = f(x) = \int_1^{x^2} \frac{\sqrt{1+u^2}}{u}\,du$$

$$F''(x) = \frac{d}{dx}\left[\int_1^{x^2} \frac{\sqrt{1+u^2}}{u}\,du\right] = \frac{\sqrt{1+(x^2)^2}}{x^2}2x = 2\frac{\sqrt{1+x^4}}{x}$$

$$F''(2) = \sqrt{1+2^4} = \sqrt{17}.$$

2. Find the arclength of $y = \frac{x^3}{6} + \frac{1}{2x}$ over the interval [1,2].

$$\begin{aligned}
L &= \int_1^2 \sqrt{1 + \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2}\,dx \\
&= \int_1^2 \sqrt{1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}}\,dx \\
&= \int_1^2 \sqrt{\frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4}}\,dx \\
&= \int_1^2 \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\,dx \\
&= \int_1^2 \frac{x^2}{2} + \frac{1}{2x^2}\,dx \\
&= \left[\frac{x^3}{6} - \frac{1}{2x}\right]_1^2 \\
&= \frac{8}{6} - \frac{1}{4} - \frac{1}{6} + \frac{1}{2} \\
&= \frac{7}{6} + \frac{1}{4} \\
&= \frac{17}{12}.
\end{aligned}$$