Quiz 1

MATH 2300-001
August 26, 2008

1. Evaluate $\lim_{x\to 0^+} x^a \ln x$ where a is any real number.

$$\begin{aligned} a>0:\ \lim_{x\to 0^+} x^a \ln x &= \lim_{x\to 0^+} \frac{\ln x}{x^{-a}} \\ &\overset{\text{L'H}}{=} \lim_{x\to 0^+} \frac{1/x}{-ax^{-a-1}} \\ &= \lim_{x\to 0^+} -\frac{1}{a}\frac{x^{a+1}}{x} \\ &= \lim_{x\to 0^+} -\frac{x^a}{a} \\ &= 0. \end{aligned}$$

$$a=0:\ \lim_{x\to 0^+} x^a \ln x = \lim_{x\to 0^+} \ln x = -\infty$$

$$a<0:\ \lim_{x\to 0^+} x^a \ln x = \left(\lim_{x\to 0^+} \frac{1}{x^{-a}}\right)\cdot\left(\lim_{x\to 0^+} \ln x\right) = (\infty)\cdot(-\infty) = -\infty$$

2. $\displaystyle\int_0^{\frac{\sqrt{2}}{4}} \frac{1}{\sqrt{1-4x^2}}\,dx =$

   Use a u-substitution of $u = 2x$.

$$\begin{aligned} \int_0^{\frac{\sqrt{2}}{4}} \frac{1}{\sqrt{1-4x^2}}\,dx &= \frac{1}{2}\int_0^{\frac{\sqrt{2}}{2}} \frac{1}{\sqrt{1-u^2}}\,du \\ &= \frac{1}{2}\left[\sin^{-1} u\right]_0^{\frac{\sqrt{2}}{2}} \\ &= \frac{1}{2}\left(\frac{\pi}{4} - 0\right) \\ &= \frac{\pi}{8}. \end{aligned}$$