Quiz 15

MATH 2300-001

December 9, 2008

1. Find the area inside the rose $r = \cos((2n+1)\theta)$.

$$
\begin{aligned}
A &= 2(2n+1) \int_0^{\frac{\pi}{2(2n+1)}} \frac{1}{2} \cos^2((2n+1)\theta)\, d\theta \\
&= \frac{2n+1}{2} \int_0^{\frac{\pi}{4n+2}} 1 + \cos((4n+2)\theta)\, d\theta \\
&= \frac{2n+1}{2} \left[\theta + \frac{1}{4n+2} \sin((4n+2)\theta) \right]_0^{\frac{\pi}{4n+2}} \\
&= \frac{2n+1}{2} \cdot \frac{\pi}{4n+2} \\
&= \frac{\pi}{4}.
\end{aligned}
$$

2. Find the total length of the spiral $r = e^{-\theta}, \quad 0 \le \theta < \infty$.

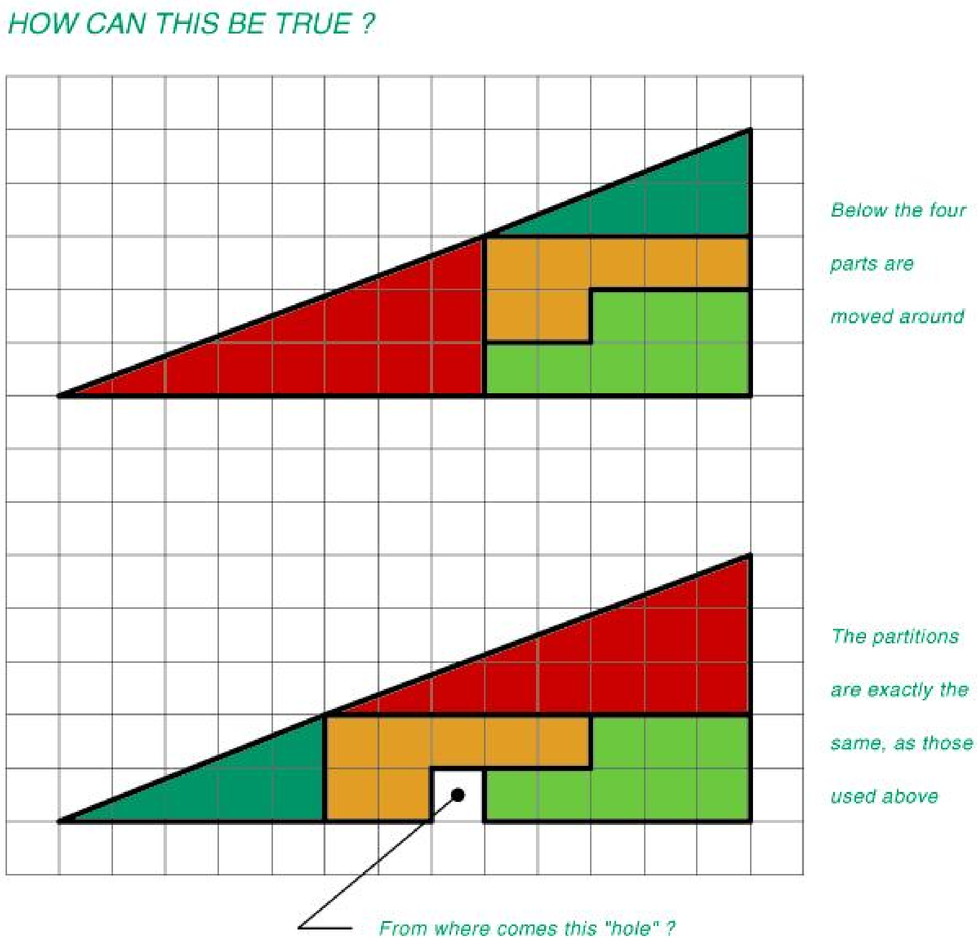
$$
\begin{aligned}
L &= \int_0^\infty \sqrt{(e^{-\theta})^2 + (-e^{-\theta})^2}\, d\theta \\
&= \int_0^\infty \sqrt{2e^{-2\theta}}\, d\theta \\
&= \sqrt{2} \int_0^\infty e^{-\theta}\, d\theta \\
&= \lim_{a\to\infty} \sqrt{2} \left[-e^{-\theta} \right]_0^a \\
&= \sqrt{2}\,(0+1) \\
&= \sqrt{2}.
\end{aligned}
$$

3. Find all values of θ where $r = e^\theta$ has a vertical or horizontal tangent line.

$$
\frac{dy}{dx} = \frac{e^\theta \sin\theta + e^\theta \cos\theta}{e^\theta \cos\theta - e^\theta \sin\theta} = -\frac{\tan\theta + 1}{\tan\theta - 1}.
$$

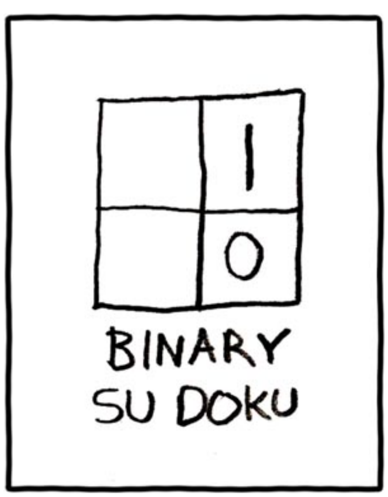
Horizontal: $\theta = \frac{3\pi}{4} + \pi k$ Vertical: $\theta = \frac{\pi}{4} + \pi k$.

4.



The first figure is not a triangle: The "slopes" of the two triangles are not the same. So, they only way this should seem wrong is if you incorrectly assume the first figure is a triangle.

5.



Do you need a solution, seriously?