Quiz 14

MATH 2300-001

December 2, 2008

1. What does the series $\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k+1}}{(2k+1)!}$ converge to?

$$\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k}}{(2k)!} = \sin \pi = 0.$$

2. $\lim_{x \to 0} \frac{x \tan^{-1}(x^3)}{1 - \cos(x^2)} =$

$$\lim_{x \to 0} \frac{x^4 - \frac{x^{10}}{3} + \frac{x^{16}}{5} - \cdots}{\frac{x^4}{2!} - \frac{x^8}{4!} + \frac{x^{12}}{6!} - \cdots} = \lim_{x \to 0} \frac{1 - \frac{x^6}{3} + \frac{x^{12}}{5} - \cdots}{\frac{1}{2} - \frac{x^4}{4!} + \frac{x^8}{6!} - \cdots} = \frac{1}{\frac{1}{2}} = 2.$$

3. (a) Find the Maclaurin series for $\ln(1 - x^4)$.
   (b) Evaluate $\int \ln(1 - x^4)\, dx$.

$$\ln(1 - x^4) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} (-x^4)^k = -\sum_{k=1}^{\infty} \frac{x^{4k}}{k}.$$

$$\int \ln(1 - x^4)\, dx = \int -\sum_{k=1}^{\infty} \frac{x^{4k}}{k}\, dx = -\sum_{k=1}^{\infty} \frac{x^{4k+1}}{k(4k+1)} + C.$$

4. Convert and simplify the following equation from Rectangular to Polar coordinates:

$$(x-a)^2+(y-a)^2=2a^2.$$

$$\begin{aligned}(x-a)^2+(y-a)^2&=2a^2\\ x^2-2ax+a^2+y^2-2ay+a^2&=2a^2\\ x^2+y^2&=2a(x+y)\\ r^2&=2ar(\cos\theta+\sin\theta)\\ r&=2a(\cos\theta+\sin\theta).\end{aligned}$$

5. Convert and simplify the following equation from Polar to Rectangular coordinates:

$$\sqrt{2}\sin\theta=1.$$

$$\begin{aligned}\sqrt{2}\sin\theta&=1\\ \sin\theta&=\frac{\sqrt{2}}{2}\\ \theta&=\frac{\pi}{4},\ \frac{3\pi}{4}\\ \tan\theta&=1,\ -1\\ \frac{y}{x}&=1,\ -1\\ \left|\frac{y}{x}\right|&=1\\ |y|&=|x|.\end{aligned}$$