NAME ______________________

College Algebra Midterm, 6/16/10

1. After an accident, police officers try to determine the approximate velocity v, in miles per hour, that a car was traveling by using the formula $v = 9\sqrt[3]{\ell}$, where ℓ is the length of the skid marks in feet.

   (a) If the skid marks were 64 feet long, how fast was the car traveling?

   We're told $\ell = 64$, so by the formula

   $$v = 9\sqrt[3]{64} = 9 \cdot 4 = \boxed{36 \text{ mph}}$$

   (b) If the car was going 63 miles per hour, how long were its skid marks?

   Now we're told $v = 63$, and we need to find ℓ. Thus,

   $$63 = 9\sqrt[3]{\ell} \iff 7 = \sqrt[3]{\ell} = \ell^{1/3} \iff \boxed{343 = 7^3 = \ell}$$

2. Compute the product $(5 + 4\sqrt{10})(1 - 2\sqrt{10})$.

   Just FOIL:

   $$\begin{aligned} 5 \cdot 1 + 5(-2\sqrt{10}) + 4\sqrt{10} \cdot 1 + 4\sqrt{10}(-2\sqrt{10}) &= 5 - 10\sqrt{10} + 4\sqrt{10} - 80 \\ &= \boxed{-75 - 6\sqrt{10}} \end{aligned}$$

3. Circle the numbers which are rational:

$\boxed{1.32}$	$\boxed{-\frac{1.245245...}{4.555...}}$	$\boxed{8^{5/3}}$	$\sqrt{-2}$
$\boxed{\sqrt[3]{64}}$	$\frac{\pi}{2}$	$\boxed{(\sqrt{5})^4}$	$0.11011101111...$

4. Compute and simplify $\dfrac{3y-4}{y^2+2y+1} - \dfrac{2y-5}{y^2+2y+1}$.

Since both fractions already have a common denominator, we can just put everything on top over it and simplify, noting that the denominator is a perfect square:

$$\begin{aligned}\frac{3y-4}{y^2+2y+1}-\frac{2y-5}{y^2+2y+1} &= \frac{3y-4-(2y-5)}{y^2+2y+1}\\ &= \frac{\cancel{y+1}}{(y+1)\cancel{(y+1)}}\\ &= \boxed{\frac{1}{y+1}}\end{aligned}$$

5. Compute and simplify $\dfrac{4p}{p^2-36}-\dfrac{2}{p-6}$.

The only thing we need to do is get a common denominator for this. Noting that $p^2-36=p^2-6^2=(p-6)(p+6)$, we see that the second fraction needs to be multiplied by $\dfrac{p+6}{p+6}$. Then the whole thing is easy to simplify:

$$\begin{aligned}\frac{4p}{p^2-36}-\frac{2}{p-6} &= \frac{4p}{(p-6)(p+6)}-\frac{2}{p-6}\cdot\frac{p+6}{p+6}\\ &= \frac{4p-2(p+6)}{(p-6)(p+6)}\\ &= \frac{4p-2p-12}{(p-6)(p+6)}\\ &= \frac{2p-12}{(p-6)(p+6)}\\ &= \frac{2\cancel{(p-6)}}{\cancel{(p-6)}(p+6)}\\ &= \boxed{\frac{2}{p+6}}\end{aligned}$$

6. Which of the properties, commutative, associative, or distributive, were used to obtain the simplification $(7+\sqrt{2})^2=51+14\sqrt{2}$?

$\boxed{\text{All three of them.}}$ Since we can only perform one operation (addition or multiplication) at a time, we have to use associativity to deal with 4 different things at some point. Distributivity is probably the most easily seen, since we have to FOIL to get anywhere, and FOILing *is* based on distributing. Finally, we'll have to move things around, or commute them, to collect like terms together. In full,

$$\begin{aligned}(7+\sqrt{2})^2 &= (7+\sqrt{2})(7+\sqrt{2}) & \\ &= (7+\sqrt{2})7+(7+\sqrt{2})\sqrt{2} & \text{distributivity}\\ &= (7^2+\sqrt{2}\cdot 7)+(7\sqrt{2}+(\sqrt{2})^2) & \text{distributivity}\\ &= (49+7\sqrt{2})+(7\sqrt{2}+2) & \text{commutativity}\\ &= 49+(7\sqrt{2}+(7\sqrt{2}+2)) & \text{associativity}\\ &= 49+((7\sqrt{2}+7\sqrt{2})+2) & \text{associativity}\\ &= 49+((7+7)\sqrt{2}+2) & \text{distributivity}\end{aligned}$$

$$\begin{aligned}
&= 49 + (14\sqrt{2} + 2) && \\
&= 49 + (2 + 14\sqrt{2}) && \text{commutativity} \\
&= (49 + 2) + 14\sqrt{2} && \text{associativity} \\
&= 51 + 14\sqrt{2} &&
\end{aligned}$$

7. Solve $2h^2 + 7h + 6 = 0$ by the most convenient method.

   This polynomial can be factored directly:

   $$2h^2 + 7h + 6 = (2h + 3)(h + 2)$$

   so by the zero product property $2h + 3 = 0$ or $h + 2 = 0$, and thus

   $$\boxed{h = -\frac{3}{2} \text{ or } h = -2}$$

   Alternatively, you could use the quadratic formula or completing the square to get the same answer.

8. Simplify the expression $\left(\frac{27p^{12}}{8q^6}\right)^{\frac{4}{3}}$.

   Notice that $27 = 3^3$ and $8 = 2^3$. Then just distribute the outer exponent 4/3 through using the power of a product and power of a quotient rules:

   $$\left(\frac{27p^{12}}{8q^6}\right)^{\frac{4}{3}} = \left(\frac{3^3p^{12}}{2^3q^6}\right)^{\frac{4}{3}} = \frac{3^{3\cdot\frac{4}{3}}p^{12\cdot\frac{4}{3}}}{2^{3\cdot\frac{4}{3}}q^{6\cdot\frac{4}{3}}} = \frac{3^4p^{16}}{2^4q^8} = \boxed{\frac{81p^{16}}{16q^8}}$$

9. Simplify the expression $\frac{(\sqrt{3}\sqrt[5]{5})^5}{2\sqrt{3}}$.

   $$\frac{(\sqrt{3}\sqrt[5]{5})^5}{2\sqrt{3}} = \frac{(3^{1/2}5^{1/5})^5}{2\cdot 3^{1/2}} = \frac{3^{5/2-1/2}5^{5/5}}{2} = \frac{9\cdot 5}{2} = \boxed{\frac{45}{2}}$$

10. Solve $5x^2 - 4x = 3$ by completing the square.

    We need to first divide by 5 to get x^2 to have a coefficient of 1:

    $$x^2 - \frac{4}{5}x = \frac{3}{5}$$

    Then add $\left(\frac{\frac{4}{5}x}{2x}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$ to both sides:

    $$x^2 - \frac{4}{5}x + \frac{4}{25} = \frac{3}{5} + \frac{4}{25}$$

    Simplify:

    $$\left(x - \frac{2}{5}\right)^2 = \frac{15 + 4}{25} = \frac{19}{25}$$

Take the square root of both sides:

$$x - \frac{2}{5} = \pm\frac{\sqrt{19}}{5}$$

Add $\frac{2}{5}$ to both sides, and you're done:

$$\boxed{x = \frac{2}{5} \pm \frac{\sqrt{19}}{5}}$$

11. Simplify $\dfrac{x^2 - 2}{x - \sqrt{2}}$.

$$\frac{x^2 - 2}{x - \sqrt{2}} = \frac{\cancel{(x - \sqrt{2})}(x + \sqrt{2})}{\cancel{x - \sqrt{2}}} = \boxed{x + \sqrt{2}}$$

12. Seven times the first of two consecutive odd integers is equal to five times the second. Find the integers.

There are two integers, so let's give them names, a and b. They're consecutive and odd, so suppose a is the first odd integer. Since the b is also odd, and since it's the next one, it must be bigger by 2. Thus,

$$b = a + 2$$

Now, the statement says seven times the first, that is $7a$, is equal to 5 times the second, that is $5b$, so we know

$$7a = 5b$$

Plugging in $b = a + 2$ into this gives

$$7a = 5(a + 2) = 5a + 10$$

Simplifying,

$$2a = 10$$

so

$$\boxed{a = 5, \qquad \text{and therefore} \qquad b = 5 + 2 = 7}$$

13. Solve the inequality $|x - 2| \leq 7$.

This is straightforward. There are two cases:

(a) Case 1: $x - 2 \leq 7$. In this case we simply add 2 to both sides and get

$$x \leq 9$$

(b) Case 2: $-(x - 2) \leq 7$. In this case, multiply both sides by -1 to get

$$x - 2 \geq -7$$

Then add 2 to both sides and you find

$$x \geq -5$$

Together, these two cases tell us that x must satisfy $-5 \leq x \leq 7$, so the solution set is

$$\boxed{\{x \mid -5 \leq x \leq 9\} \qquad \text{or in interval notation} \qquad [-5, 9]}$$

14. Solve $\sqrt{x-2} - \sqrt{2x} = -2$.

This one isn't so straightforward. It takes two steps. First, we need to get rid of the roots, then we need to solve the result for x. Let's start with the first task: add $\sqrt{2x}$ to both sides so that we have a root on each side:

$$\sqrt{x-2} = \sqrt{2x} - 2$$

Then square both sides:

$$\begin{aligned} x - 2 &= (\sqrt{2x} - 2)^2 \\ &= (\sqrt{2x})^2 - 2\sqrt{2x} \cdot 2 + 2^2 \\ &= 2x - 4\sqrt{2x} + 4 \end{aligned}$$

Now, we need to isolate $\sqrt{2x}$ so we could square it and so get rid of the square root sign: add $4\sqrt{2x}$ to both sides, and subtract $x - 2$ from both sides:

$$4\sqrt{2x} = x + 6$$

Square both sides:

$$32x = 16 \cdot 2x = (x+6)^2 = x^2 + 12x + 36$$

We're now done with the first step, we've gotten rid of the root signs. Now we need to solve for x. Subtract $32x$ from both sides:

$$0 = x^2 - 20x + 36$$

This polynomial factors:

$$0 = (x - 18)(x - 2)$$

so

$$\boxed{x = 18 \;\text{ or }\; 2}$$

15. Solve $x^{3/5} + 17 = 9$.

Put all the numbers on the right side:

$$x^{3/5} = -8$$

Then take the 5/3 power of both sides, so that you get x all alone, and simplify:

$$x = (x^{3/5})^{5/3} = (-8)^{5/3} = [(-2)^3]^{5/3} = (-2)^5 = \boxed{-32}$$

Extra Credit: Factor $x^{12} - 2x^6 + 1$.

$$\begin{aligned}
x^{12} - 2x^6 + 1 &= (x^6 - 1)^2 \\
&= \Big[(x^3 - 1)(x^3 + 1)\Big]^2 \\
&= \Big[(x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)\Big]^2 \\
&= \boxed{(x - 1)^2(x^2 + x + 1)^2(x + 1)^2(x^2 - x + 1)^2}
\end{aligned}$$

Alternatively, you could have used the difference of cubes formula in the second line, but then it's not easy to see the factorization of $x^4 + x^2 + 1$:

$$\begin{aligned}
x^{12} - 2x^6 + 1 &= (x^6 - 1)^2 \\
&= \Big[(x^2 - 1)(x^4 + x^2 + 1)\Big]^2 \\
&= \Big[(x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1)\Big]^2 \\
&= \boxed{(x - 1)^2(x^2 + x + 1)^2(x + 1)^2(x^2 - x + 1)^2}
\end{aligned}$$

The reason $x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$ is because it's actually a difference of squares, but that isn't obvious:

$$x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2 = (x^2 + 1)^2 - x^2 = (x^2 + 1 - x)(x^2 + 1 + x)$$