ex [redacted]. Infinite sequence of trials, each trial having

p = prob. of success

$1-p$ = prob. of failure

(a) What is the prob. that there is $\underbrace{\text{at least one success in first } n \text{ trials}}_{\text{call this } E}$?

A: The prob. of 0 successes is

$(1-p)^n \Leftarrow$ (prod. bec. indep. events)

So $\boxed{P(E) = 1-(1-p)^n}$

(b) What is the prob. of exactly $k \leq n$ successes in n trials?

A: $$\boxed{\begin{aligned} &P(\text{exactly } k \text{ successes}) \\ &\quad = \binom{n}{k} p^k (1-p)^{n-k} \end{aligned}}$$

each of k successes has prob. p

\# ways to get k successes in n

$n-k$ failures each w/ prob. $1-p$

* product bec. indep.

(c) What is the prob. that <u>all</u> trials result in success?

<u>A:</u> Let E_i = success on ith trial. Then,

$$P\left(\bigcap_{i=1}^{\infty} E_i\right) = P\left(\lim_{n\to\infty} \bigcap_{i=1}^{n} E_i\right)$$

* bec. $E_1, E_1 \cap E_2, \ldots, \bigcap_{i=1}^{n} E_i, \ldots$ is a decreasing seq.

$$= \lim_{n\to\infty} P\left(\bigcap_{i=1}^{n} E_i\right)$$

$$= \lim_{n\to\infty} p^n$$

$$= \boxed{\begin{cases} 0, & \text{if } 0 \le p < 1 \\ 1, & \text{if } p = 1 \end{cases}}$$

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ex. 2 A card is selected at random from a deck of 52. Let

E = card is an ace

F = card is a spade

Then,

$P(E) = \frac{1}{13}$ (13 denominations 2, 3, ..., Ace)

$P(F) = \frac{1}{4}$ (4 suits)

and since $E \cap F$ = card is an ace of spades,

$P(E \cap F) = \frac{1}{52}$

so

$$P(E \cap F) = \frac{1}{52}$$
$$= \frac{1}{13} \cdot \frac{1}{4}$$
$$= P(E)P(F)$$

Remark: since E and F are indep., so are E and F^c where F^c = ~~all~~ of the other 3 suits. $\boxtimes$

The events are independent, i.e. denomination is independent of suit. $\boxtimes$

ex. 3 Throw two fair dice and let

dependent

$E_1 =$ sum is 6

$E_2 =$ sum is 7

$F =$ first die is 4 independent!

Then,

$E_1 \cap F =$ 1st die is 4
2nd die is 2

$E_2 \cap F =$ 1st die is 4
2nd die is 3

and

$P(E_1) = \frac{5}{36}$

$P(F) = \frac{1}{6}$ (2nd die has 6 options)

$P(E_2) = \frac{6}{36} = \frac{1}{6}$

$P(E_1 \cap F) = \frac{1}{36}$

$P(E_2 \cap F) = \frac{1}{36}$

$\Longrightarrow \quad P(E_1 \cap F) = \frac{1}{36} \neq \frac{5}{36} \cdot \frac{1}{6} = P(E_1)P(F)$

$P(E_2 \cap F) = \frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6} = P(E_2)P(F)$

$\boxtimes$

ex. 4 Two fair dice are thrown,

E = sum is 7

F = 1st die is 4

G = 2nd die is 3

We know from ex. 3 that E & F are indep.
Show that E is indep. of G, but that
E is not indep. of $F \cap G$.

A:

$$\left.\begin{aligned} P(E \cap G) &= \frac{1}{36} \\ P(E) &= \frac{6}{36} = \frac{1}{6} \\ P(G) &= \frac{1}{6} \end{aligned}\right\} \text{ so } P(E \cap G) = P(E)P(G)$$

and

$$P(E \mid F \cap G) = 1 \quad \left(= \frac{1/36}{1/36} = \frac{P(E \cap (F \cap G))}{P(F \cap G)} \right)$$

$$\left.\begin{aligned} P(E \cap F \cap G) &= \frac{1}{36} \\ P(E) &= \frac{1}{6} \\ P(F \cap G) &= \frac{1}{36} \end{aligned}\right\} \quad P(E \cap (F \cap G)) \neq P(E)P(F \cap G)$$

$\boxtimes$