Homework 7

Color Scheme: Blue problems are graded, orange and red are not. In fact, don't turn in orange and red ones, but *you should try to solve them for yourselves, as exercises.*

Graded	Important	Routine

New groups for this homework:

(1) Yiting Song, Elliot Spears, Aaron Mutchler
(2) Rod Jafari, Baraka Kombe-Jarvis, Michelle Maclennan
(3) Alexa Graffeo, Nathan Lowe, Jade Vanausdall
(4) Tristan Hanna, Alexander Straiting, Ahmed Alenezi
(5) Aaron Hong, John Vander Dussen, Yi Xu
(6) Brady Itkin, Bryan Nelson

Problems:

- (1) Prove that if $A \in \mathrm{GL}(n, \mathbb{R})$, then A will take any basis $\beta = (\mathbf{b}_1, \ldots, \mathbf{b}_n)$ for $\mathbb{R}^n$ to some other basis $A\beta := (A\mathbf{b}_1, \ldots, A\mathbf{b}_n)$ for $\mathbb{R}^n$. (*Compare with Exercise 16 in Section 4.7.*)
  (2) Prove the converse, that if $A \in M_n(\mathbb{R})$ will take any basis β for $\mathbb{R}^n$ to another basis $A\beta$, then $A \in \mathrm{GL}(n, \mathbb{R})$. [Hint: Consider $\ker A$.] (*Also, compare with Exercise 15 in Section 4.7.*)
  (3) Consider the standard basis $\beta = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ for $\mathbb{R}^3$ and the mathrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 6 & 0 & 0 \end{pmatrix}$.
  Use the previous exercises to show that the column vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ of A form a basis for $\mathbb{R}^3$. Do the rows $\vec{A}_1, \vec{A}_2, \vec{A}_3$ also form a basis?
- Section 5.1: 17, 19, 6, 7, 13, 14, 16, 30ab
- Section 4.7: 2b, 14, 6, 8ab
- Section 5.2: 7ab, 14 (this is the Riesz Representation Theorem), 3de, 20, 21, 22
- Section 5.3: 6, 10, 8, 14, 16
- Section 5.4: 4ab, 5ab, 8a