Homework 6

Color Scheme: Blue problems are graded, orange and red are not. In fact, don't turn in orange and red ones, but *you should try to solve them for yourselves, as exercises.*

Graded	Important	Routine

New groups for this homework:

(1) Yiting Song, Elliot Spears, Aaron Mutchler
(2) Rod Jafari, Baraka Kombe-Jarvis, Michelle Maclennan
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(5) Aaron Hong, John Vander Dussen, Yi Xu
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Problems:

- Let $p(x) = -1 + x^2$, $q(x) = x + x^2 \in \mathcal{P}_2$.
  (1) Prove that the dimension of $\mathcal{P}_2$ is 3 by showing that $\rho = \{1,\ x,\ x^2\}$ is a basis for $\mathcal{P}_2$.
  (2) Show that the set $S = \{p, q\}$ is linearly independent in $\mathcal{P}_2$.
  (3) Show that S is not a spanning set by exhibiting a polynomial $r(x) \in \mathcal{P}_2 \setminus \operatorname{span}(S)$.
  (4) Extend S to a basis $\beta = \{p, q, r\}$ for $\mathcal{P}_2$, and make sure to prove all your claims!
- This is a modified version of problem 5 in Section 4.4. Let
$$S = \left\{ \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 3 & 2 \\ -6 & 1 \end{pmatrix}, \begin{pmatrix} 4 & -1 \\ -5 & 2 \end{pmatrix}, \begin{pmatrix} 3 & -3 \\ 0 & 0 \end{pmatrix} \right\} \subseteq M_2(\mathbb{R})$$
  and let us label the four matrices $A_1,\ A_2,\ A_3,\ A_4$, and write O for the zero matrix $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.
  (5) Show that S is linearly dependent by finding scalars $a_1, a_2, a_3, a_4 \in \mathbb{R}$, not all 0, such that $\sum_{i=1}^{4} a_i A_i = O$. Use these to write A_4 as a linear combination of A_1, A_2 and A_3.
  (6) Find a smaller subset $\beta \subseteq S$ of S which *is* linearly independent and satisfies $\operatorname{span}(\beta) = \operatorname{span}(S)$. This is a basis for $\operatorname{span}(S)$. What is the dimension of $\operatorname{span}(S)$?
- Section 4.4: 10 (In fact, show that more generally $\mathbf{v}_1, \ldots, \mathbf{v}_n \in \mathbb{R}^n$ are linearly independent iff $\det\begin{pmatrix} \mathbf{v}_1 & \cdots & \mathbf{v}_n \end{pmatrix} \neq 0$), where the vectors are considered as columns.), 16, 20b, 7ab, 23
- Section 4.5: 6abc, 7, 14, 15acd, 4bc, 8ab,
- Section 4.6: 14ab, 20, 19, 21