Linear Algebra Practice Midterm 1
Spring 2019

1. Let
$$A = \begin{pmatrix} -2 & 3 & -3 & -1 \\ 4 & 1 & 13 & -5 \end{pmatrix}$$
and consider the homogeneous system $A\mathbf{x} = \mathbf{0}$, where $\mathbf{x} \in \mathbb{R}^4$ and $\mathbf{0} \in \mathbb{R}^2$.

   (a) Compute $\operatorname{rref}\big(A \mid \mathbf{0}\big)$.

   (b) Identify the pivot columns $\mathbf{b}_j$ in $B = \operatorname{rref}\big(A \mid \mathbf{0}\big)$.

(c) Write the remaining, non-pivot, columns of B as linear combinations of the pivot columns you identified in part (b).

(d) Returning to the *original* augmented matrix $\left(A \,|\, \mathbf{0}\right)$, show directly that the same dependency relations hold among the columns of A (e.g. if you found that $\mathbf{b}_4 = 5\mathbf{b}_1 + 2\mathbf{b}_3$, then you would demonstrate, starting with the RHS, that $\mathbf{a}_4 = 5\mathbf{a}_1 + 2\mathbf{a}_3$).

(e) Parametrize the free variables, those corresponding to the non-pivot columns of B, and use the form of B to solve for the dependent variables in terms of the independent.

(f) Use the previous part to write *all possible* solutions, namely the null space $N(A)$ of A, as a span of column vectors.

(g) Use the previous step to find two different solutions of $A\mathbf{x} = \mathbf{0}$. Plug them in to demonstrate that they are, in fact, solutions.

2. Let $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 0 \\ -4 \\ -3 \end{pmatrix}$, $\mathbf{w} = \begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} \in \mathbb{R}^3$. Are these three vectors linearly independent? If not, write $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

3. **True or False**: If a matrix A has trivial null space, $N(A) = \{\mathbf{0}\}$, then as a function A is one-to-one.

4. **True or False**: If $A \in M_{3,5}(\mathbb{R})$, then as a function A cannot be one-to-one.

5. **True or False**: The matrix A in problem #1 above is onto.

6. **True or False**: A square matrix A is invertible (i.e. both one-to-one and onto as a function) if and only if the columns of A are linearly independent.

7. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \in M_2(\mathbb{R})$ and $B = \begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{pmatrix} \in M_{2,3}(\mathbb{R})$. Compute $A^{-1}B$.