Quiz 9

Definition 0.1 A square matrix $A \in M_n(\mathbb{R})$ is called **symmetric** if A equals its own transpose,

$$A = A^T$$

where we recall that the transpose A^T of A is the matrix with columns the old rows of A,

$$A^T = (\vec{A}_1 \quad \cdots \quad \vec{A}_n)$$

1. Show that for any (not necessarily square) $A \in M_{m,n}(\mathbb{R})$, the product matrix AA^T is a square $(m \times m)$ symmetric matrix. Simply demonstrate the key identity, that the transpose of AA^T equals AA^T.

$$(AA^T)^T = ((A^T)^T) A^T = AA^T$$

2. Show that for any square matrix $A \in M_n(\mathbb{R})$, not necessarily symmetric, the sum $A + A^T$ is symmetric. Again, just show the key identity in the definition of symmetric.

$$(A + A^T)^T = A^T + A^{TT} = A^T + A$$

$$= A + A^T$$

3. Let $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \in M_2(\mathbb{R})$.

(a) Compute A^T.

$$A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

(b) Compute AA^T. It should be symmetric.

$$AA^T = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 11 \\ 11 & 25 \end{pmatrix}$$

which is indeed symmetric (it equals its own transpose).

(c) Compute $A + A^T$. It should be symmetric.

$$A + A^T = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 5 & 8 \end{pmatrix}$$

which is symmetric