Quiz 9

Definition 0.1 A square matrix $A \in M_n(\mathbb{R})$ is called **symmetric** if A equals its own transpose,

$$A = A^T$$

where we recall that the transpose A^T of A is the matrix with columns the old rows of A,

$$A^T = \begin{pmatrix} \vec{A}_1 & \cdots & \vec{A}_n \end{pmatrix}$$

1. Show that for any (not necessarily square) $A \in M_{m,n}(\mathbb{R})$, the product matrix AA^T is a square ($m \times m$) symmetric matrix. Simply demonstrate the key identity, that the transpose of AA^T equals AA^T.

2. Show that for any square matrix $A \in M_n(\mathbb{R})$, not necessarily symmetric, the sum $A + A^T$ is symmetric. Again, just show the key identity in the definition of symmetric.

3. Let $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \in M_2(\mathbb{R})$.

   (a) Compute A^T.

   (b) Compute AA^T. It should be symmetric.

   (c) Compute $A + A^T$. It should be symmetric.