Quiz 8

Let

$$A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix} \in M_{3,5}(\mathbb{R})$$

and suppose we plugged this into Wolfram and got

$$\text{rref}(A) = \begin{pmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Let $B = \text{rref}(A)$, so that we could call the columns and rows $\mathbf{b}_j$ and $\vec{B}_i$, respectively.

1. The pivot columns of B are $\mathbf{b}_1$ and $\mathbf{b}_3$. Write each of the other (nonpivot) columns of B as linear combinations of $\mathbf{b}_1$ and $\mathbf{b}_3$:

   (a) $\mathbf{b}_2 =$ $= \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix} = -2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = (-2)\vec{b}_1 \boxed{= (-2)\vec{b}_1 + 0\vec{b}_3}$

   (b) $\mathbf{b}_4 =$ $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = (-1)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \boxed{= (-1)\vec{b}_1 + 2\vec{b}_3}$

   (c) $\mathbf{b}_5 =$ $\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (-2)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \boxed{= 3\vec{b}_1 + (-2)\vec{b}_3}$

2. Show that in the original matrix A the columns $\mathbf{a}_2$, $\mathbf{a}_4$ and $\mathbf{a}_5$ may be written in exactly the same linear combination of $\mathbf{a}_1$ and $\mathbf{a}_3$:

   (a) Demonstrate that the same $a, b \in \mathbb{R}$ as in #1(a) also satisfy $\mathbf{a}_2 = a\mathbf{a}_1 + b\mathbf{a}_3$

   $$(-2)\vec{a}_1 + 0\vec{a}_2 = (-2)\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ -4 \end{pmatrix} = \vec{a}_2 \checkmark$$

   (b) Do the same for $\mathbf{a}_4$.

   $$(-1)\vec{a}_1 + 2\vec{a}_3 = (-1)\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3-2 \\ -1+4 \\ -2+10 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} = \vec{a}_4 \checkmark$$

   (c) Repeat for $\mathbf{a}_5$.

   $$3\vec{a}_1 + (-2)\vec{a}_3 = 3\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} + (-2)\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -9+2 \\ 3-4 \\ 6-10 \end{pmatrix} = \begin{pmatrix} -7 \\ -1 \\ -4 \end{pmatrix} = \vec{a}_5 \checkmark$$