Quiz 8

Let

$$A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix} \in M_{3,5}(\mathbb{R})$$

and suppose we plugged this into Wolfram and got

$$\mathrm{rref}(A) = \begin{pmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Let $B = \mathrm{rref}(A)$, so that we could call the columns and rows $\mathbf{b}_j$ and $\vec{B}_i$, respectively.

1. The pivot columns of B are $\mathbf{b}_1$ and $\mathbf{b}_3$. Write each of the other (nonpivot) columns of B as linear combinations of $\mathbf{b}_1$ and $\mathbf{b}_3$:
   (a) $\mathbf{b}_2 =$

   (b) $\mathbf{b}_4 =$

   (c) $\mathbf{b}_5 =$

2. Show that in the original matrix A the columns $\mathbf{a}_2$, $\mathbf{a}_4$ and $\mathbf{a}_5$ may be written in exactly the same linear combination of $\mathbf{a}_1$ and $\mathbf{a}_3$:
   (a) Demonstrate that the same $a, b \in \mathbb{R}$ as in #1(a) also satisfy $\mathbf{a}_2 = a\mathbf{a}_1 + b\mathbf{a}_3$

   (b) Do the same for $\mathbf{a}_4$.

   (c) Repeat for $\mathbf{a}_5$.