# Quiz 7

1. Let $A = \begin{pmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{pmatrix} \in M_3(\mathbb{R})$ and let $\mathbf{u} = \begin{pmatrix} -7 \\ 3 \\ 2 \end{pmatrix} \in \mathbb{R}^3$. Is $\mathbf{u}$ in the null space $N(A)$ of $A$?

Yes: $A\vec{u} = \begin{pmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{pmatrix}\begin{pmatrix} -7 \\ 3 \\ 2 \end{pmatrix}$

$= -7\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 0 \\ -5 \end{pmatrix} + 2\begin{pmatrix} 5 \\ 7 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -7 - 3 + 10 \\ -14 + 0 + 14 \\ 21 - 15 - 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \vec{0}.$

2. **True or False:** An $m \times n$ matrix $A = (\mathbf{a}_1 \ \cdots \ \mathbf{a}_n)$ has trivial null space, $N(A) = \{\mathbf{0}\}$, iff ("if and only if") the columns of $A$ are linearly independent in $\mathbb{R}^m$.

True: $N(A) = \{\vec{0}\} \iff$ "$A\vec{x} = \vec{0} \Rightarrow \vec{x} = \vec{0}$"

$\iff$ "$x_1\vec{a}_1 + \cdots + x_n\vec{a}_n = \vec{0}$

$\Rightarrow x_1 = \cdots = x_n = 0$"

$\iff \{\vec{a}_1, \ldots, \vec{a}_n\}$ is lin. indep.

3. Let $A \in M_{m,n}(A)$ and let $\mathbf{v} \in \mathbb{R}^n$, **True or False:** $\mathbf{v}$ is in the null space $N(A)$ of $A$ iff $\mathbf{v}$ is orthogonal to each of the rows $\vec{A}_i$ of $A$ ($i = 1, \ldots, m$), in the sense of the dot product,

$$\vec{A}_i \cdot \mathbf{v} = 0$$

True: $\vec{v} \in N(A) \iff A\vec{v} = \vec{0}$

$\iff \begin{pmatrix} \vec{A}_1 \cdot \vec{v} \\ \vdots \\ \vec{A}_m \cdot \vec{v} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

$\iff \vec{A}_i \cdot \vec{v} = 0$ for all $i$.