

Quiz 6

1. An $m \times n$ matrix

$$A = \begin{pmatrix} | & | & \cdots & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \\ | & | & \cdots & | \end{pmatrix}$$

can be viewed as a function from \mathbb{R}^n to \mathbb{R}^m , sending $\mathbf{x} = \langle x_1, \dots, x_n \rangle \in \mathbb{R}^n$ to $A\mathbf{x} = \sum_{i=1}^n x_i \mathbf{a}_i = \mathbf{b} \in \mathbb{R}^m$.

- (a) True or False: If $m < n$, then as a function A cannot be one-to-one.
- (b) True or False: If $m > n$, then as a function A cannot be onto.
- (c) True or False: A is one-to-one iff ('if and only if') the columns of A are linearly independent.
- (d) True or False: A is onto iff the columns of A span \mathbb{R}^n .
- (e) True or False: If $m = n$, then A , necessarily a square matrix, is invertible (both one-to-one and onto) iff $\mathbb{R}^n = \text{span}(\mathbf{a}_1, \dots, \mathbf{a}_n)$.