Quiz 3

1. Recall that, as shown in class, the homogeneous system $A\mathbf{x} = \mathbf{0}$ given by

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

becomes, in row-reduced echelon form,

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and that therefore the 3 planes corresponding to the 3 rows of the original system, whose normal vectors are the rows of A,

$$\vec{A}_1 = \langle 1, 2, 3 \rangle$$
$$\vec{A}_2 = \langle 4, 5, 6 \rangle$$
$$\vec{A}_3 = \langle 7, 8, 9 \rangle$$

must all lie in the same plane. *Show this*, namely show that $\vec{A}_3$ lies in the span of $\vec{A}_1$ and $\vec{A}_2$, or in other words **find constants $a, b \in \mathbb{R}$ such that**

$$a\vec{A}_1 + b\vec{A}_2 = \vec{A}_3$$

You may find it easier to work with column vectors rather than row vectors.

$a\vec{A}_1 + b\vec{A}_2 = \vec{A}_3$ means

$$a\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + b\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \qquad (*)$$

and we could rephrase this as a matrix product, whose definition is precisely $(*)$:

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix},$$ i.e. $A\vec{x} = \vec{b}$ with $A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$, $\vec{b}$

But then, by Wolfram, $\text{ref}(A|\vec{b}) = \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$, which says $\boxed{\begin{array}{l} a = -1 \\ b = 2 \end{array}}$.

Not being content with what Wolfram tells me, I want to check it myself:

$$(-1)\vec{A}_1 + 2\vec{A}_2 = (-1)\begin{pmatrix}1\\2\\3\end{pmatrix} + 2\begin{pmatrix}4\\5\\6\end{pmatrix}$$

$$= \begin{pmatrix}-1\\-2\\-3\end{pmatrix} + \begin{pmatrix}8\\10\\12\end{pmatrix}$$

$$= \begin{pmatrix}-1+8\\-2+10\\-3+12\end{pmatrix}$$

$$= \begin{pmatrix}7\\8\\9\end{pmatrix}$$

$$= \vec{A}_3$$ boom!