

## Quiz 2

1. Solve the following system

$$\begin{aligned} x - 2y + z &= 0 \\ 2y - 8z &= 0 \\ 5x - 5z &= 0 \end{aligned}$$

Let us, for variety, use the augmented matrix

$$(A|\vec{0}) = \left( \begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 0 \\ 5 & 0 & -5 & 0 \end{array} \right)$$

$$\xrightarrow{-5I+III} \left( \begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 0 \\ 0 & 10 & -10 & 0 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} \frac{1}{2}II \\ \frac{1}{10}III \end{array}} \left( \begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right)$$

$$\xrightarrow{-II+III} \left( \begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 3 & 0 \end{array} \right)$$

$$\xrightarrow{\frac{1}{3}III} \left( \begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

← divide through to simplify things so I would make mistakes later

This is just a schematic representation of

$$\begin{array}{l} x - 2y + z = 0 \quad I \\ y - 4z = 0 \quad II \\ \boxed{z = 0} \quad III \end{array}$$

We've got  $z$ !

Option 1 Back substitute: plug  $z=0$   
into II:  $y - 4 \cdot \underset{z}{0} = 0 \Rightarrow \boxed{y=0}$ .

Then plug  $y=z=0$  into I:

$$x - \underset{y}{2 \cdot 0} + \underset{z}{0} = 0 \Rightarrow \boxed{x=0}$$

Thus,  $x=y=z=0$  is the soln.

Option 2: Bring  $(A|\vec{0})$  all the way to rref  $(A|\vec{0})$ :  
we pick up where we left off:

$$\left( \begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) \xrightarrow[\text{-III+I}]{\text{4III+II}} \left( \begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

$$\xrightarrow{\text{2II+I}} \boxed{\left( \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right)} \leftarrow = \text{rref}(A|\vec{0})$$

$$\boxed{\begin{array}{l} x = 0 \\ y = 0 \\ z = 0 \end{array}}$$