Quiz 14

Consider the standard and a non-standard basis for $\mathbb{R}^2$,

$$\sigma = (\mathbf{e}_1, \mathbf{e}_2) = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right), \quad \beta = (\mathbf{b}_1, \mathbf{b}_2) = \left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right),$$

and define the β-coordinate representation of a vector $\mathbf{x} = \binom{3}{1} \in \mathbb{R}^2$ to be

$$[\mathbf{x}]_\beta = \begin{pmatrix} a \\ b \end{pmatrix}, \quad \text{whenever} \quad \mathbf{x} = a\mathbf{b}_1 + b\mathbf{b}_2$$

and the β-coordinate representation of a matrix $A = \begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix} \in M_2(\mathbb{R})$ to be

$$[A]_\beta = \big([A\mathbf{b}_1]_\beta \ \ [A\mathbf{b}_2]_\beta \big)$$

The change-of-coordinates matrix changing β-coordinates to γ-coordinates is

$$M_{\beta,\gamma} = \Big([\mathbf{b}_1]_\gamma \ \ [\mathbf{b}_2]_\gamma \Big)$$

You may use the following facts in your calculations:

- $[\mathbf{x}]_\sigma = \mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^2$
- $[\mathbf{x}]_\gamma = M_{\beta,\gamma}[\mathbf{x}]_\beta$
- $M_{\beta,\sigma} = \big([\mathbf{b}_1]_\sigma \ \ [\mathbf{b}_2]_\sigma \big) = \big(\mathbf{b}_1 \ \ \mathbf{b}_2 \big)$
- $M_{\gamma,\beta} = M_{\beta,\gamma}^{-1}$
- $M_{\beta,\delta} = M_{\gamma,\delta} M_{\beta,\gamma}$ for any three bases β, γ, δ.

1. Find $[\mathbf{x}]_\beta$.

2. Find $[A]_\beta$.