Quiz 12

Consider the standard and two non-standard bases for $\mathbb{R}^2$,

$$\sigma = (\mathbf{e}_1, \mathbf{e}_2) = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right), \quad \beta = (\mathbf{b}_1, \mathbf{b}_2) = \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right), \quad \gamma = (\mathbf{c}_1, \mathbf{c}_2) = \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)$$

and define the β-coordinate representation of a vector $\mathbf{x} \in \mathbb{R}^2$ to be

$$[\mathbf{x}]_\beta = \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{means} \quad \mathbf{x} = a\mathbf{b}_1 + b\mathbf{b}_2$$

and similarly for $[\mathbf{x}]_\gamma$ (of course, $[\mathbf{x}]_\sigma = \mathbf{x}$).

1. Find $M_{\beta,\sigma} = \big([\mathbf{b}_1]_\sigma, [\mathbf{b}_2]_\sigma\big)$ and $M_{\sigma,\beta} = \big([\mathbf{e}_1]_\beta, [\mathbf{e}_2]_\beta\big) = M_{\beta,\sigma}^{-1}$.

2. Find $M_{\gamma,\sigma} = \big([\mathbf{c}_1]_\sigma, [\mathbf{c}_2]_\sigma\big)$ and $M_{\sigma,\gamma} = \big([\mathbf{e}_1]_\gamma, [\mathbf{e}_2]_\gamma\big) = M_{\gamma,\sigma}^{-1}$.

3. Find $M_{\beta,\gamma} = \big([\mathbf{b}_1]_\gamma, [\mathbf{b}_2]_\gamma\big)$ and $M_{\gamma,\beta} = \big([\mathbf{c}_1]_\beta, [\mathbf{c}_2]_\beta\big) = M_{\beta,\gamma}^{-1}$.

4. Verify that the components of the vector $[\mathbf{c}_1]_\beta$ are the coefficients c_i of $\mathbf{c}_1 = c_1\mathbf{b}_1 + c_2\mathbf{b}_2$ and the components of the vector $[\mathbf{c}_2]_\beta$ are the coefficients d_i of $\mathbf{c}_2 = d_1\mathbf{b}_1 + d_2\mathbf{b}_2$.