Quiz 10

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \in M_2(\mathbb{R})$ and $B = \begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{pmatrix} \in M_{2,3}(\mathbb{R})$.

1. Compute AB.

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 7 & -8 & 9 \\ 13 & -16 & 27 \end{pmatrix}}$$

2. Compute $B^T A$.

$$B^T A = \begin{pmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \boxed{\begin{pmatrix} 11 & 14 \\ -13 & -16 \\ 15 & 18 \end{pmatrix}}$$

3. Compute A^{-1}.

$$A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} = \boxed{-\frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}}$$

4. Is B^{-1} possible as a function? Key words to think about here are 'one-to-one,' 'onto,' 'null space' and 'range.'

B^{-1} is impossible, B's null space must be nontrivial, so B is not 1-1.