1. If $f$ is the function whose graph is shown, let $h(x)=f(f(x))$ and $g(x)=f\left(x^{2}\right)$. Use the graph of $f$ to estimate the value of each derivative.
(a) $h^{\prime}(2)$
(b) $g^{\prime}(2)$

2. Under certain circumstances a rumor spreads according to the equation

$$
p(t)=\frac{1}{1+a e^{-k t}}
$$

where $p(t)$ is the proportion of the population that knows the rumor at time $t$ and $a$ and $k$ are positive constants. (In Calculus 2 we will see that this is a reasonable equation for $p(t)$.)
(a) Find $\lim _{t \rightarrow \infty} p(t)$
(b) Find the rate of spread of the rumor.
(c) Graph $p$ for the case $a=10, k=0.5$ with $t$ measured in hours. use the graph to estimate how long it will take for $80 \%$ of the population to hear the rumor.
3. If $x y+e^{y}=e$, find the value of $y^{\prime \prime}$ at the point where $x=0$.
4. Two curves are orthogonal if their tangent lines are perpendicular at each point of intersection. Show that the given families of curves are orthogonal trajectories of each other, that is, every curve in one family is orthogonal to every curve in the other family. Sketch both families of curves on the same axes.

$$
y=a x^{3} \text { and } x^{2}+3 y^{2}=b
$$

5. A particle moves along the curve $y=\sqrt{1+x^{3}}$. As it reaches the point $(2,3)$, the $y$-coordinate is increasing at a rate of $4 \mathrm{~cm} / \mathrm{s}$. How fast is the $x$-coordinate of the point changing at that instant?
6. A street light is mounted at the top of a 15 -ft-tall pole. A man 6 ft tall walks away from the pole with a speed of $5 \mathrm{ft} / \mathrm{s}$ along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?
(a) What quantities are given in the problem?
(b) What is the unknown?
(c) Draw a picture of the situation for any time $t$.
(d) Write an equation that relates the quantities.
(e) Finish solving the problem.
7. Water is leaking out of an inverted conical tank at a rate of $10,000 \mathrm{~cm}^{3} / \mathrm{min}$ at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m . If the water level is rising at a rate of $20 \mathrm{~cm} / \mathrm{min}$ when the height of the water is 2 m , find the rate at which water is being pumped into the tank.
8. The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips of the hands changing at one o'clock?

## Optional Challenge Problems

Sketch the circles $x^{2}+y^{2}=1$ and $(x-3)^{2}+y^{2}=4$. There is a line with positive slope that is tangent to both circles. Determine the points at which this tangent line touches the circle.

