MATH 2300 - Homework 3

Instructions: This homework is due on Friday, June 22^{nd} . You may work with other students, however each person is responsible for writing their own solutions. Please write the names of any students who helped you.

1. Determine if the following improper integral converges or diverges. If convergent, evaluate the integral, or if that is not possible, give an upper bound for the integral.

$$\int_{2}^{\infty} \frac{1}{x^3 - 2x} \, dx$$

- 2. The rate r at which people get sick during an epidemic of the flu can be approximated by $r = 1000te^{-0.5t}$ where r is measured in people per day and t is the number of days since the epidemic started.
 - (a) When are people getting sick the fastest?
 - (b) Using the answer from part (a), how many people are sick by that day.
 - (c) How many people get sick altogether?
 - (d) Give an upper bound of the day by which half the people are sick.
 - (e) Give a lower bound for the day by which all but 10 people are sick.
- 3. When both bounds of an improper integral were finite, we chose to define

$$\int_{-\infty}^{\infty} f(x) \, dx = \int_{-\infty}^{c} f(x) \, dx + \int_{c}^{\infty} f(x) \, dx.$$

Show that divergence or convergence of the integral does not depend on the choice of c.

- 4. The Great Pyramid of Egypt has a square base with 755 ft edges and has height 410 ft. Calculate the volume of the pyramid using vertical slices, and then using horizontal slices to check your answer.
- 5. Using both horizontal and vertical slices, find the area bounded by f(x) and g(x) given:
 - (a) $f(x) = -x^2 + 6$, g(x) = 4|x|.
 - (b) $f(x) = x^2, g(x) = \sqrt{x}.$
- 6. Find the volume of the "horn" created stacking disks of radius $\frac{1}{x}$ as x goes from 1 to ∞ .