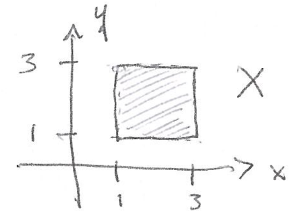
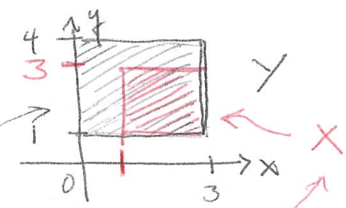
Hw 2 - evens

Sec. 1.5: (6) $X = [1,3] \times [1,3]$



$Y = [0,3] \times [1,4]$



Notice, $X \subseteq Y$:

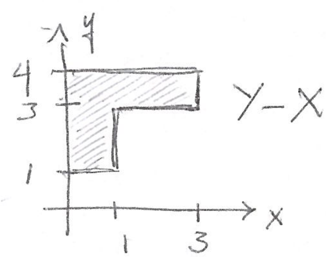
so

$$X \cup Y = Y$$

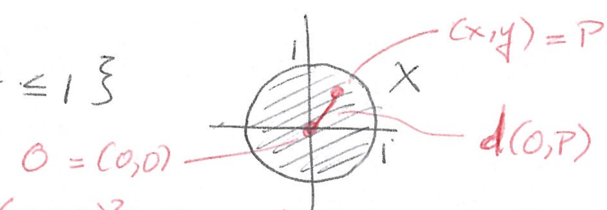
$$X \cap Y = X$$

$$X - Y = \emptyset$$

$Y - X$:

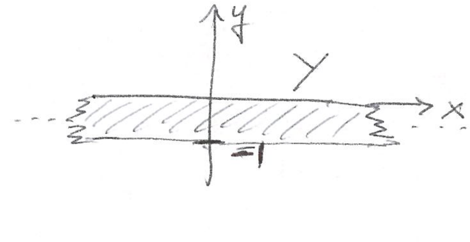


(8) $X = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$

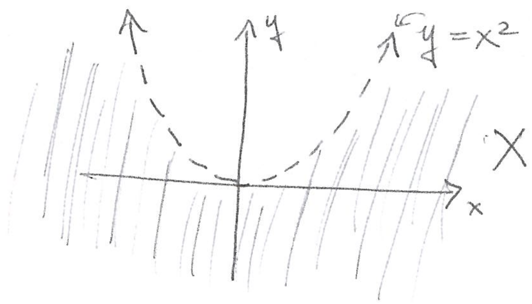


$*$
$$\begin{aligned} x^2+y^2 &= (x-0)^2+(y-0)^2 \\ &= d(O,P)^2 \\ &\leq 1 \end{aligned}$$

$Y = \{(x,y) \in \mathbb{R}^2 \mid -1 \leq y \leq 0\}$

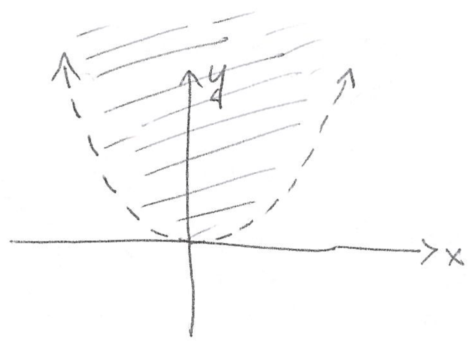


Sec. 1.6 (6) $X = \{(x,y) \in \mathbb{R}^2 \mid y < x^2\}$

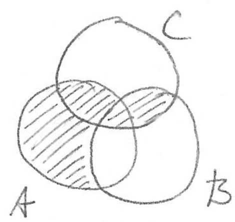


vs.

$\overline{X}$

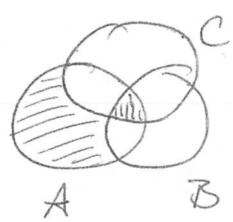


Sec. 1.7 (12)



$(B \cap C) \cup (A - B)$

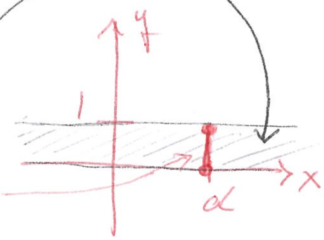
(14)



$(A \cap B \cap C) \cup (A - (B \cup C))$

Sec. 1.8

(8)(a) $$\underbrace{\bigcup_{\alpha\in\mathbb{R}} \overset{x}{\{\alpha\}}\times \overset{y}{[0,1]}}_{=\{(x,y)\,\mid\, x=\alpha,\ 0\le y\le 1\}} = \boxed{\mathbb{R}\times[0,1]}$$



(b) $\bigcap_{\alpha\in\mathbb{R}} \{\alpha\}\times[0,1] = \boxed{\emptyset}$ bec. if $\alpha\neq\beta$,

$$(\{\alpha\}\times[0,1])\cap(\{\beta\}\times[0,1]) = \emptyset$$

for $\alpha\neq\beta \Rightarrow x=\alpha$ <u>and</u> $x=\beta$ is impossible.

(12) $$\bigcap_{\alpha\in I} A_\alpha = \bigcup_{\alpha\in I} A_\alpha \Longrightarrow \boxed{\text{all } A_\alpha = A_\beta = A}$$

<u>pf:</u> Let $\alpha\neq\beta$ in I. Clearly $\bigcap_{\alpha\in I} A_\alpha \subseteq A_\alpha$,

while $A_\alpha \subseteq \bigcup_{\alpha\in I} A_\alpha = \bigcap_{\alpha\in I} A_\alpha$

any x in here is in every A_α, including ours

$x \Rightarrow x\in$ one of these

Therefore, $A_\alpha = \bigcap_{\alpha\in I} A_\alpha$

& similarly $A_\beta = \bigcap_{\alpha\in I} A_\alpha = A_\alpha$.

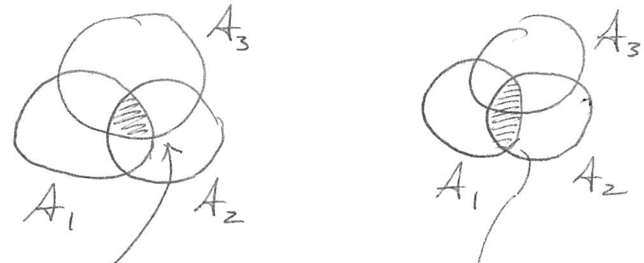
QED

(14) $\emptyset \neq J \subseteq I \implies \bigcap_{\alpha \in I} A_\alpha \subseteq \bigcap_{\alpha \in J} A_\alpha$.

ex. $J = \{1, 2\} \implies \bigcap_{\alpha \in J} A_\alpha = A_1 \cap A_2$

$I = \{1, 2, 3\}$

$\implies \bigcap_{\alpha \in I} A_\alpha = A_1 \cap A_2 \cap A_3$



$A_1 \cap A_2 \cap A_3 \subseteq A_1 \cap A_2$

pf: $x \in \bigcap_{\alpha \in I} A_\alpha \iff x \in A_\alpha$ for <u>each</u> $\alpha \in I$

$\implies$ since $\alpha \in J \subseteq I \implies \alpha \in I$,

$x \in A_\alpha$ for all $\alpha \in J$

$\implies x \in \bigcap_{\alpha \in J} A_\alpha$

Sec. 2.1

(8) $\mathbb{N} = \mathbb{N}$ means $\mathbb{N} \subseteq \mathbb{N}$ & $\mathbb{N} \supseteq \mathbb{N}$.

Alternatively, $x \in \mathbb{N} \Rightarrow x \in \mathbb{N}$

$\Longrightarrow \mathbb{N} \subseteq \mathbb{N}$

$\Longrightarrow \mathbb{N} \in \mathcal{P}(\mathbb{N})$.

So $\boxed{\text{False}}$.

(10) $(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R})$

$= \mathbb{N} \times \mathbb{N}$

is $\boxed{\text{True}}$

pf: $(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R})$

$$= \{(x,y) \in \mathbb{R}^2 \mid x \in \mathbb{R},\ y \in \mathbb{N}\} \cap \{(x,y) \in \mathbb{R}^2 \mid x \in \mathbb{N},\ y \in \mathbb{R}\}$$

$$= \{(x,y) \mid x \in \underbrace{\mathbb{R} \cap \mathbb{N}}_{=\mathbb{N}},\ y \in \underbrace{\mathbb{N} \cap \mathbb{R}}_{=\mathbb{N}}\}$$

$$= \mathbb{N} \times \mathbb{N}$$

(12) If the integer x is a multiple of 7, then it is divisible by 7.

$\boxed{\text{True}}$

pf: $x = 7k$ means both x is a multiple of 7 and is divisible by 7, since

$$\mathbb{Z} \ni x = 7k \iff \frac{x}{7} = k \in \mathbb{Z}.$$