MATH 2300 - Homework 2

Instructions: This homework is due on Friday, June 15$^{\text{th}}$. You may work with other students, however each person is responsible for writing their own solutions. Please write the names of any students who helped you.

1. Compute the following integrals using partial fractions:

   (a) $\displaystyle\int \frac{2x}{x^3 - x^2 + x - 1}\, dx$

   (b) $\displaystyle\int \frac{3}{s^4 - 1}\, ds$

   (c) $\displaystyle\int \frac{x^4 + 12x^3 + 15x^2 + 25x + 11}{x^3 + 12x^2 + 11x}\, dx$

2. Compute the integral $\displaystyle\int \frac{1 + e^x}{1 - e^x}\, dx$ in three different ways:

   (a) Use the substitution $u = 1 - e^x$.

   (b) Use the substitution $u = e^x$. [Hint: Multiply the integrand by e^x/e^x.]

   (c) Write $\dfrac{1 + e^x}{1 - e^x} = \dfrac{1}{1 - e^x} + \dfrac{e^x}{1 - e^x}$ and perform two separate integrals using appropriate substitutions.

3. Compute the following integrals using your favorite method:

   (a) $\displaystyle\int \frac{1}{\sqrt{x^2 + 10}}\, dx$

   (b) $\displaystyle\int \frac{x}{\sqrt{x^2 + 10}}\, dx$

   (c) $\displaystyle\int \frac{x^2}{\sqrt{x^2 + 10}}\, dx$

4. For the definite integral $\displaystyle\int_1^3 e^{x^2}\, dx$, calculate the following:

   (a) LEFT(4)
   (b) RIGHT(4)
   (c) TRAP(4)
   (d) MID(4)

   Which give an overestimate and which give an underestimate? Explain.

5. (a) Find the exact value of $\int_0^4 e^x \, dx$.

   (b) For the integral in (a) find LEFT(2), RIGHT(2), TRAP(2), MID(2), SIMP(2), and compute the error for each.

   (c) For the integral in (a) find LEFT(4), RIGHT(4), TRAP(4), MID(4), SIMP(4), and compute the error for each.

   (d) For the integral in (a) find LEFT(8), RIGHT(8), TRAP(8), MID(8), SIMP(8), and compute the error for each.

   (e) Compute the error ratios LEFT(2)/LEFT(4) and LEFT(4)/LEFT(8), and similarly with RIGHT, TRAP, MID, and SIMP.

   You may use a computer to do this if you wish. For example, you can alter this simple MATLAB code (it will also work with OCTAVE, the open source analog), which computes the left and right Riemann sums LEFT(n) and RIGHT(n) of the integral $\int_1^3 \sin^2(x) \, dx$. Just copy the text into text file and name it integral.m, say, then run it in MATLAB or OCTAVE.

6. Determine if the following integrals converge or diverge. If an integral converges, compute the value of the integral. You may appeal to the domninance of one function over another or to L'Hôpital's rule.

   (a) $\int_0^\infty x e^{-x^2} \, dx$

   (b) $\int_4^2 0 \frac{1}{x^2 - 16} \, dx$

   (c) $\int_7^\infty \frac{1}{\sqrt{t-5}} \, dt$

7. For what values of p does the integral $\int_e^\infty x^p \ln x \, dx$ converge? What is the value of the integral when it converges?