Hw 1 – Evens

Sec. 1.1

(A2) $\{3x+2 \mid x \in \mathbb{Z}\} = \boxed{\{\ldots, \overset{x=-2}{-4}, \overset{x=-1}{-1}, \overset{x=0}{2}, \overset{x=1}{5}, \ldots\}}$

(A8) $\{x \in \mathbb{R} \mid x^3 + 5x^2 = -6x\} = \boxed{\{-3, -2, 0\}}$

since $x^3 + 5x^2 = -6x$

$$\iff x^3 + 5x^2 + 6x = 0$$

$$\iff x(x^2 + 5x + 6) = 0$$

$$\iff x(x+3)(x+2) = 0$$

$$\iff x = -3, -2, 0$$

(A16) $\underbrace{\{6a+2b \mid a, b \in \mathbb{Z}\}}_{\text{call this } X} = \boxed{2\mathbb{Z} = \{\ldots, -4, -2, 0, 2, 4, \ldots\}}$

since, first,

$X \subseteq 2\mathbb{Z}$ because if $x = 6a+2b \in X$,

then $x = 2\underbrace{(3a+b)}_{\in \mathbb{Z}} \in 2\mathbb{Z}$

But also $2\mathbb{Z} \subseteq X$, because $Y \overset{\text{def}}{:=} \{3a+b \mid a, b \in \mathbb{Z}$

equals $\mathbb{Z}$: certainly $Y \subseteq \mathbb{Z}$ because $3, a, b \in \mathbb{Z}$ $\Longrightarrow$ $3a+b \in \mathbb{Z}$. But to see that $\mathbb{Z} \subseteq Y$, let

$$n \in \mathbb{Z}$$

and let us find $a, b \in \mathbb{Z}$ s.t. $n = 3a+b$. Well,

$$a = n$$
$$b = -2n$$

are certainly integers and

$$3a + b = 3n + (-2n) = n$$

Thus, $Y = \{3a+b \mid a, b \in \mathbb{Z}\} = \mathbb{Z}$, so

$$\boxed{X = 2Y = 2\mathbb{Z}}.$$

<u>QED</u>

(B22) $\{3, 6, 11, 18, 27, 38, \ldots\} = \boxed{\{x_n \mid x_n = 2 + \sum_{k=0}^{n} 2k+1\},\ n \in \mathbb{Z}}$

$\underbrace{6-3}=\boxed{3}$

$\underbrace{11-6}=\boxed{5}$

$\underbrace{18-11}=\boxed{7}$

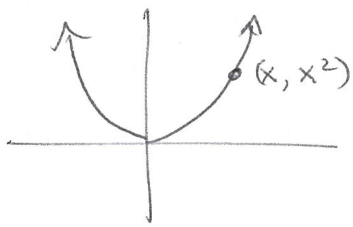
$\underbrace{27-18}=\boxed{9}$ etc.

differences are of the form $2k+1$, $k \in \mathbb{N}$.

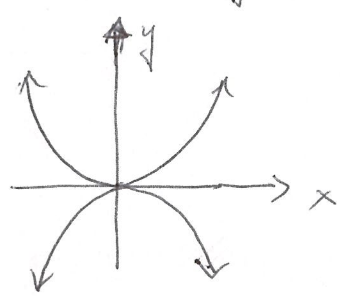
$x_1 = 2 + (2\cdot 0+1$
$= 3$

$x_2 = 2 + (2\cdot 0+1)$
$+ (2\cdot 1+1)$
$= 6$

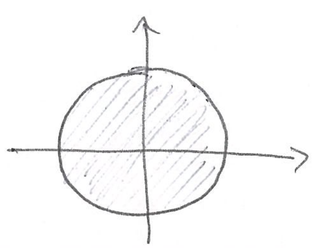
(D44) $\{(x, x^2) \mid x \in \mathbb{R}\}$



(D52) $\{(x,y) \in \mathbb{R}^2 \mid (y-x^2)(y+x^2) = 0\}$



(D46) $\{(x,y) \mid x, y \in \mathbb{R},\ x^2+y^2 \leq 1\}$

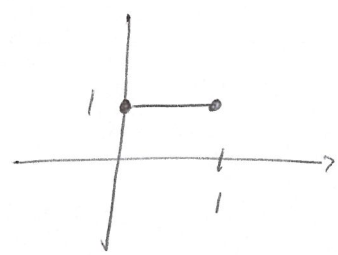


Sec. 1.2

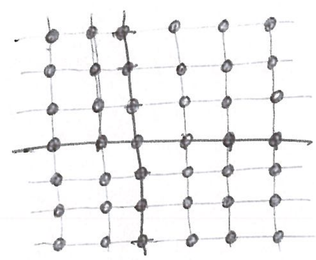
(A8) $\{0,1\}^4 = \{(0,0,0,0),\ (0,1,0,0),\ (0,0,1,0),$
$(0,0,0,1),\ (0,1,1,0),\ (0,1,0,1),$
$(0,0,1,1),\ (0,1,1,1),$

$(1,0,0,0),\ (1,1,0,0),\ (1,0,1,0),$
$(1,0,0,1),\ (1,1,1,0),\ (1,1,0,1)$
$(1,0,1,1),\ (1,1,1,1)\ \}$

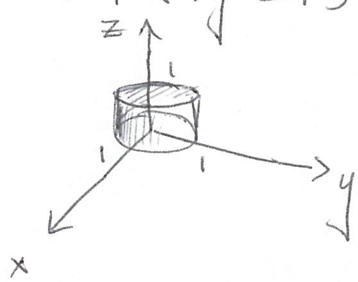
(B16) $\underset{\substack{\rotatebox{90}{$\in$} \\ x}}{[0,1]} \times \underset{\substack{\rotatebox{90}{$\in$} \\ y}}{\{1\}}$



(B18) $\mathbb{Z} \times \mathbb{Z}$



(B20) $\{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \le 1\} \times \underset{\substack{\rotatebox{90}{$\in$} \\ z}}{[0,1]} \subseteq \mathbb{R}^3$



Sec. 1.3

(10) $\{X \subseteq \mathbb{N} \mid |X| \leq 1\} = \boxed{\{\emptyset, \{1\}, \{2\}, \ldots\}}$

(14) $\boxed{F,}$ $\mathbb{R}^2 \times \underset{\substack{\in \\ \mathbb{Z}}}{\{0\}} \subseteq \mathbb{R}^3$

Sec. 1.4

(A2) $\mathcal{P}(\{1,2,3,4\}) = \boxed{\begin{array}{l} \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \\ \{1,2\}, \{1,3\}, \{1,4\}, \\ \{2,3\}, \{2,4\}, \{3,4\}, \\ \{1,2,3\}, \{1,2,4\}, \\ \{1,3,4\}, \{2,3,4\}, \\ \{1,2,3,4\}\} \end{array}}$

(A8) $\mathcal{P}(\{1,2\} \times \{3\}) = \boxed{\begin{array}{l} \{\emptyset, \{1,2\} \times \{3\}, \\ \{1\} \times \{3\}, \{2\} \times \{3\}\} \end{array}}$

(B14) $|\mathcal{P}(\mathcal{P}(A))| = 2^{|\mathcal{P}(A)|} = 2^{2^{|A|}} = \boxed{2^{2^m}}$

(B16) $|\mathcal{P}(A) \times \mathcal{P}(B)| = |\mathcal{P}(A)||\mathcal{P}(B)|$

$= 2^{|A|} \cdot 2^{|B|}$

$= \boxed{2^{m+n}}$