1. Find the limit. Use l'Hôpital's Rule if appropriate. If there is a more elementary method, consider using it. If l'Hôpital's Rule does not apply, explain why.

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$$

2. Use l'Hôpital's Rule to help find the asymptotes of f. Then use them, together with the information from f' and f'', to sketch the graph. Check your work with a graphing device or program.

$$f(x) = xe^{-x^2}$$

3. Prove that

$$\lim_{x\to\infty}\frac{\ln(x)}{x^p} = 0$$

for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x.

4. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account is

$$v = \frac{mg}{c}(1 - e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant. (In Calculus 2 we will be able to deduce this equation from the assumption that the air resistance is proportional to the speed of the object; c is the proportionality constant.)

(a) Calculate $\lim_{t\to\infty} v$. What is the meaning of this limit?

(b) For fixed t, use l'Hôpital's Rule to calculate $\lim_{c\to 0^+} v$. What can you conclude about the velocity of a falling object in a vacuum?

5. If f' is continuous, $f(2) = 0$, and $f'(2) = 7$, evaluate

$$\lim_{x\to 0}\frac{f(2+3x)+f(2+5x)}{x}$$

Optional Challenge Problem

This problem will help improve your algebra skills.

Graph the function $f(x) = (x-2)^{1/3}x^{2/3}$. Include calculation of the first and second derivatives and full analysis of the increasing/decreasing behavior, local extrema, concavity and inflection points, and the end behavior.