MATH 2300-001
Final Exam Review

1. $\frac{d}{dx}\left[\cosh(\operatorname{sech}(2x))\right] =$

2. $\int \frac{x^3}{x^2-2x+1}\,dx =$

3. $\int_0^1 x^4 e^{-x}\,dx =$

4. $\int_0^1 e^{x^2}\,dx =$

5. $\int_{\pi/4}^{\pi/3} \tan^5 x \sec^4 x\,dx =$

6. $\int_1^\infty \frac{1}{x^2\sqrt{x^2-1}}\,dx =$

7. Find the general solution to the differential equation $(1+x)y' + y = \sqrt{x}$.

8. Solve the initial value problem $y'' - 2\sqrt{2}y' + 2y = 0, y(0) = 0, y'(0) = 5$.

9. Determine if the sequence converges. If it does, find its limit.

   (a) $\left\{\frac{(-4)^n}{n!}\right\}_{n=0}^{\infty}$

   (b) $\left\{\frac{1-(-1)^n}{\sqrt{n}}\right\}_{n=1}^{\infty}$

10. Determine if the series diverges, converges conditionally, or converges absolutely. If the series converges, find its sum.

    (a) $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+3)}$

    (b) $\sum_{k=1}^{\infty} \frac{k+1}{k^2+(-1)^k k+1}$

    (c) $\sum_{k=1}^{\infty} \frac{10^k}{k4^{2k+1}}$

    (d) $\sum_{k=0}^{\infty} \frac{(-5)^k}{k!}$

11. Find the interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{(k+1)x^{2k-1}}{3^k}$

12. Find the Taylor series for $f(x) = \frac{1}{1-x}$ expanded around $x = 5$.

13. Approximate $\cos(\frac{1}{2})$ to 2 decimal places using the Maclaurin series for $\cos(x)$.

14. Using series, prove that $\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$.

15. Convert the polar equation $\sin(2\theta) = 1$ to Cartesian coordinates.

16. Find the length of $r = \theta$ from $\theta = 0$ to $\theta = \pi$.

17. Find the area between the loops of the limaçon $r = 1 + \sqrt{2}\sin\theta$.