

MATH 2300: CALCULUS 2

4:30 - 7:00 pm, Sat. Dec. 13, 2008

FINAL

I have neither given nor received aid on this exam.

Name: _____

Check one below!

- | | |
|---|---|
| <input type="radio"/> 001 P. NEWBERRY(9AM) | <input type="radio"/> 004 S. LIMBURG(12PM) |
| <input type="radio"/> 002 E. WITTENBORN (10AM) | <input type="radio"/> 005 M. STACKPOLE (2PM) |
| <input type="radio"/> 003 R. KRIEGER (11AM) | <input type="radio"/> 006 S. IH (8AM) |

If you have a question raise your hand and remain seated. In order to receive credit your answer must be **complete, logical, legible, and correct**. Your score also depends on clarity. Show all of your work, and give adequate explanations. No shown work even with the correct final answer, no points! Only one answer to each problem! In case of two different answers to one problem, the lower score will be chosen!

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	15 pts	
2	15 pts	
3	20 pts	
4	20 pts	
5	35 pts	
6	20 pts	
7	20 pts	
8	15 pts	
9	20 pts	
10	20 pts	
TOTAL	200 pts	

1. (15 points) Evaluate the integrals below. Fully justify your answers below.

(a) (7 points)

$$\int \ln x \, dx.$$

(b) (8 points)

$$\int_2^{\infty} \frac{1}{x(\ln x)^2} \, dx.$$

2. (15 points) Solve the following differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0,$$

where $y(0) = 1$ and $y'(0) = -1$.

3. (20 points) Fully justify your answers to the questions below.

(a) (10 points) Tell if the sequence

$$\left\{ \frac{\sin^2 k}{k} \right\}_{k=1}^{\infty}$$

converges. If so, find its limit. Otherwise, tell why not.

(b) (10 points) Tell if the series

$$\sum_{k=1}^{\infty} k^2 e^{-k}$$

converges.

4. (20 points) Find the radius and the interval of convergence of

$$\sum_{k=1}^{\infty} \frac{x^k}{k}.$$

5. (35 points) Fully justify your answers to the questions below.

(a) (12 points) Use integration by parts to evaluate the integral

$$\int_0^1 xe^x dx.$$

(b) (8 points) Find the Maclaurin series for xe^x by any method.

(c) **(10 points)** Express the integral $\int_0^1 xe^x dx$ as an infinite series.

(d) **(5 points)** What does the series you found in part (c) converge to ?

6. (20 points) Evaluate the integral

$$\int \frac{dx}{(1-x^2)^{3/2}}.$$

7. (20 points) Fully justify your answers to the questions below.

(a) (6 points) Consider the point $(-3, \sqrt{3})$ given in rectangular coordinates. Find a polar coordinate of the point.

(b) (6 points) Consider the point $(-2, -\frac{5\pi}{6})$ given in polar coordinates. Find the rectangular coordinate of the point.

(c) (8 points) Let $r = 4 \sin \theta$, where $\theta \in [0, 2\pi]$. Convert this polar coordinate equation to a rectangular coordinate equation, and use the resulting rectangular equation to draw its graph.

8. (15 points) Find all the values of $\theta \in [0, 2\pi]$ where $r = 2 + \sin \theta$ has a horizontal tangent line.

9. (20 points) Find the arc length of $r = \sin^4\left(\frac{\theta}{4}\right)$ where θ runs over $[0, 2\pi]$.

10. (20 points) Fully justify your answers to the questions below.

(a) (8 points) Find the graph of the polar equation

$$r = 1 + \cos \theta.$$

- (b) **(12 points)** Find the area of the region bounded by the graph of $r = 1 + \cos \theta$ in the first quadrant.