

## Descartes

### I. The Background

I will mostly follow Gilson's 'The Unity of Philosophical Experience' here, and in what follows.

From part I, 'The Medieval Experiment,' I single out the following important points:

(1) Peter Abailard or Abelard (1079 - 1142)

was a French medieval scholastic philosopher whom Gilson singles out for his initiation of the analysis of universals: 'What is a class of things, or in other words, what is

The essence of universality? (Gibson, p. 4)

Facts:

- (1) individual things in the world  
are particular

- (2) classes of universals are generalities

- (3) generalities are conceptual, and  
hence originate in the mind

- (4) generalities nevertheless apply to  
individual objects, in the  
sense of explaining their "nature"  
& behavior

What's the connection? A universal/concept,  
says Abelard, is that which can be predicated  
(i.e. said ~~(to)~~) of several individuals. But what  
is the nature of such predication? What  
is, e.g. "man"?

"Man, for instance, is a universal because the term can be applied <sup>to</sup> [or predicated of] every individual man." — Gilson, p. 10

Logically, there is a problem with the notion of "presence in" an individual of a quality or concept or universal, if by that ~~is~~ is meant "entirely present in" because individuals are themselves "part of all humans or whatever," & so cannot singly contain the totality of a thing, nor can a totality be divided among several individuals without losing its totality.

Conclusion: the notion of universals leads to this contradiction, therefore one must abandon the (Platonic) idea that universals exist.

Universals, Abelard concludes, can only be predicated (= said of) words! Universal names have no object, mean nothing and refer to nothing.

Conclusion: universals are not things, but  
"states" or "conditions", a "way of being",  
and must be only mental//psychological  
constructs, & in ~~themselves~~ <sup>unknowable</sup> some power of God  
or divinity. <sup>themselves</sup>

I.e. we have no clue, and will never  
have a clue — scepticism.

Upshot: philosophy, reduced to logic, is dysfunctional.  
Islamic ~~philosophers~~ upped such scepticism, too  
(Gazali & Avicenna), against philosophy &  
for religion. — mysticism.  
cf. Maimonides

Combined with the overexaggerated modesty  
of Bonaventura & others - ~~pity~~.

Abelard had said that universals must at least  
be mental constructs, ~~the imagination's creations~~  
or the like, Ockham (1287-1347), an English  
Franciscan friar & scholastic philosopher, rejects  
even this, saying that universals & forms cannot  
correspond to anything at all: only individual things  
exist - nominalism, and antirealism

Ockham's razor: one should not account for the  
existence of an empirically given thing by imagining,  
behind & beyond it, another thing whose  
hypothetical existence cannot be verified.

All of this is accompanied by what Gilson calls psychologism, which he links to 'positivism' and 'scientism'. Ockham opposed even the existence of causality as a reality, contending that in fact it was habituation by repeated experience of regularity and observation of patterns:

"[...] he [mistakes] the empirical description of our ways of knowing for a correct description of reality. That is psychologism..."

— Gilson, p. 71

*"How do you know?" rather than  
"What do you know?" and "What  
is the logical or evidential reason  
for that knowledge?"*

This state of affairs effected a split between philosophy and theology, with theology reigning supreme:

"The easiest way to show that philosophy cannot prove anything against religion was to show that it cannot prove anything at all. Hence the current of metaphysical scepticism that runs through the late middle ages & whose presence can still be observed as late as the 17th-c." — Gilson p. 77

quote bottom p. 79

escapes: moralism ~~&~~ mysticism (Renaissance)  
(humanism)

quote p. 90

II. ~~Descartes' Larger Influence~~ - His mathematical physics, and more generally his mathematization of philosophy and knowledge.

Between Descartes and medieval scepticism lies the Renaissance, and it is this which is Descartes' immediate background. And not just the Renaissance, but the breakup of the Catholic church during the (Protestant) Reformation, which itself came on the heels of the discovery of the Americas and the quick work made of their conquest by Spain (a Catholic country). Also, Ottoman conquest of Byzantium (1453)!

The Renaissance in Italy (1300's - 1600s, roughly) is a complex, multifaceted reaction to the previous late medieval scholastic logical stalemate. It seems to me best to view it as a movement, humanism, whose disunity is visible by its political, literary, and philosophical factionalism.

There are the literary figures: Dante, Petrarch, Boccaccio, Tasso, etc. For example, Petrarch ~~never~~ took philosophical scepticism at all advised by good manners (it's impolite to lodge either complaints against or arguments for any metaphysics: arguments are tritious & unpleasant!)

But there are also the philosophical figures:

Lorenzo de' Medici's Florence hosted the

Florentine Academy, a Platonic school

founded in 1438-1439, when the Byzantine

 Platonic scholar Gemistus Pletho passed

and Nicholas of Cusa & Giordano Bruno through of life a mark. Marsilio Ficino

and Pico della Mirandola, are seminal representatives

of this school: generally speaking, we may say

this school attempted a mystical unification of

(neo-) Platonism, Christian Theology, and

the inheritance of classical Humanism (esp.

through Roman interpretations (esp. Cicero)).

Platonism, however, was not the dominant position taken in Italy. Platonism was symbolic of the cultural élite of Florence, whereas in Venice, for example, and Padua & Bologna, ~~the~~ Aristotelians held sway ~~strongly~~. Pomponazzi & Zabarella are the major exponents, but there are many others (Poliziano, Salutati, Porta).

The Renaissance Aristotelians were rebelling against the Late Medieval scholastics (esp. in France & Britain) by emphasizing the other, non-logical aspects of Aristotle's philosophy: e.g. rhetoric, grammar, and especially ethics (see Grassi, 'Renaissance Humanism', and Cassirer, Kristeller et al., 'The Renaissance

## Philosophy of man.')

Sum-total: Descartes' reaction was stimulated largely by Renaissance humanism's transformation of the philosophical stalemate (the inability of logic alone to make progress without metaphysics) into an artistic and cultural moralism whose message of duty was to abstain from dogmatic philosophical arguments (seemingly grounded only in logic) and to promote rather ethical and cultural values. The emphasis changed from pursuits of logical coherence to pursuits of vision, ingenuity/inventiveness, creativity, imagination.

Montaigne The straw that broke the camel's back, for Descartes, was Montaigne. His was a French interpretation of Renaissance humanism: dogmas come and go, says Montaigne, but people never learn the lesson, that dogmas (= philosophical positions, esp. systems) are not direct access to reality, and that raising philosophical controversies based on one's own dogmatic position is not only mistaken as regards truth, but is in fact produce only ~~rude~~ rude behavior. A civilized, noble character puts manners and esp. modesty above "being philosophically right," and this means abstension from philosophical system-building. Customs & traditions are more trustworthy than personal philosophies.

Needless to say, Montaigne would never get a job at CERN with such an attitude. Montaigne, and with him many other Renaissance humanists, was unscientific in his worldview. His wisdom was an ethical wisdom, like Socrates', and unlike Aristotle's & Euclid's.

Descartes, who ~~disliked~~ the traditional humanistic education he received at his elite private school at La Flèche, was inclined rather towards math and science. So as the Renaissance was coming to an end, & the Protestant Reformation solidifying its rebellion against the more humanistic Catholic church,

Descartes took inspiration from two clearly successful, philosophically positive works:

- (1) ancient Greek mathematics, esp. the post-Euclidean period of late antiquity
  - (2) Galileo Galilei (1564-1642), who had initiated modern mathematical physics as well as refined experimental method, arriving at striking new results:
- (i) discovery of the concept of density
  - (ii) discovery of gravitational acceleration (the only known non-impact form of a "force"), apparently uniform
  - (iii) relativity of motion (Newton's 1st Law, the Law of Inertia)

among others,  
see Galileo  
supplement

Descartes' early work in math was so successful and imaginative, that it inspired him to try to generalize The mathematical method to all of science, beginning with physics. Biology was soon to follow, and with it medicine.

So great was Descartes' confidence that he stopped doing math altogether in order to first make medicine and biology deductive sciences. After this was done, he could undoubtedly prolong his life for another 100 years, & have plenty of time to spare for math, physics, and anything else needing ~~some~~ mopping up.

As to Galileo, see the appended notes I wrote up in 2018.

### III. Descartes' Math.:

Descartes' first, and most successful achievement, was in math, and this while he was young, likely in his early 20s, though his 'Geometry' was not published until 1637.

Descartes solved Pappus' problem, but his methods really divide into 3:

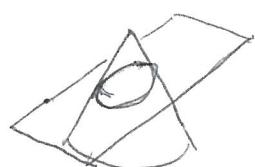
- (1) coordinates
- (2) unification of geometry & algebra w/ variables, esp. involving the order: now we define geometric objects as solutions to algebraic eqns
- (3) classification: algebraic (= "geometric") curves vs. transcendental (= "mechanical")

Concerning point (3), the classification of curves in the Euclidean plane, this is part of the Pappus problem which Descartes solved and generalized, and to which I turn next.

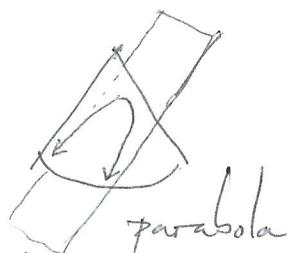
Pappus of Alexandria lived c. 290–350 AD, approx., i.e. some 500+ years after Euclid, and his immediate mathematical predecessor was Diophantus of Alexandria (c. 205–290 AD), and we may summarize their respective achievements with Wells & Boyer (p. 7): 'Pappus was a geometer and Diophantus [not exactly a contemporary] was an algebraist... but it required a mathematician who

was familiar with both... to take the next step, and that turned out to be Descartes, some 300 years later.

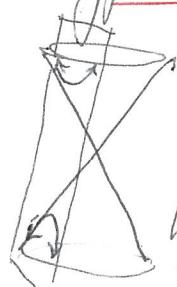
Here is what happened: Pappus was concerned with how to classify the different curves available in Greek mathematics up to that point. First, there was Euclid's <sup>Elements</sup>, in which all curves were constructed, and their relations to algebra was somewhat hidden behind these constructions (e.g. Pythagorean Thm.). After Euclid came Apollonius of Perga (c. 262-190 BC), who discovered the conic sections (ellipse, parabola, hyperbola)



ellipse



parabola



hyperbola

Apollonius discovered the algebraic equations these curves must have in relation to certain distances measured from the focii of auxiliary fixed lines. These are the theorems he discovered & proved.

Moreover, Apollonius, & seemingly Archimedes before him, developed ~~a~~ similar coordinate systems, with the important difference that, not having "negative magnitudes," & hence negative numbers, the x- and y- axes did not have "negative parts."

\* With Descartes' analytic geometry, Apollonius' results become a simple theorem: given 3 lines, say,

$$A_1x + B_1y + C_1 = 0 \quad \text{and} \quad 3 \text{ angles}$$

$$A_2x + B_2y + C_2 = 0$$

$$A_3x + B_3y + C_3 = 0 \quad \theta_1, \theta_2, \theta_3$$

The points  $(x, y)$  satisfying

"the square of the distance to one line  
is proportional to the products of the  
distances to the remaining lines,"

i.e.

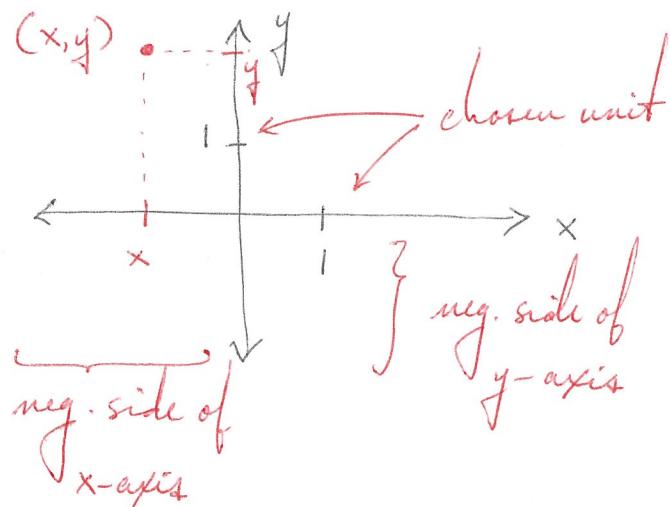
$$\frac{(A_1x + B_1y + C_1)^2}{(A_1^2 + B_1^2) \sin \theta} = k \frac{(A_2x + B_2y + C_2)}{\sqrt{A_2^2 + B_2^2} \sin \theta_2} \cdot \frac{(A_3x + B_3y + C_3)}{\sqrt{A_3^2 + B_3^2} \sin \theta_3}$$

is the solution to a quadratic equation in the plane. By linear algebra methods such an equation can be shown to be a conic section (by an appropriate change-of-coordinates).

In fact, this is what Descartes himself proved in his Geometry (1637), among other things.

## Cartesian coordinate system

In fact, after introducing his new coordinate system, in which the x- and y-axes have "negative sides",



all of the issues in Greek mathematics became transparent, though new issues were to arise.

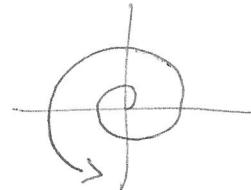
Constructible Numbers First of all, the relationship between constructible points (once an origin  $O = (0,0)$  is fixed, those points  $P$  which can be connected by a line segment  $OP$  of length  $r = |OP|$ ) ~~especially~~ and algebra is precisely  $\mathbb{Q} + \text{square roots}$  (or, equiv.  $\mathbb{Q} + i\mathbb{Q}$  w/ roots of  $\text{rationality}$ )

Once the transference from constructibility to the axioms (algebraic axioms) of  $\mathbb{Q} + \text{roots}$ . was effected, the relationship between different types of curves and different algebraic properties or expressions became an object of investigation.

(This is why Galois' proof that  $\deg. \geq 5$  polynomials are generally not solvable by radicals is significant: not all algebraic equations are exactly solvable — a setback for Cartesians).

Descartes' enthusiasm turned everything upside-down:  
he now defines curves by giving their algebraic equations, e.g. the conchoid is the solution to

$$(x-a)^2(x^2-y^2) = b^2x^2$$



With this apparatus in hand, Descartes set about solving & generalizing the ancient Pappus problem, and thereby initiated algebraic geometry.

As a consequence of the connection bet.  $\mathbb{Q} + \text{roots}$  with constructible numbers and the resulting algebraization of some of the known Greek curves, Descartes completed Pappus' original classification:

These lead  
to algebraic  
geometry

{ • any curve representable as a polynomial equation we shall consider "good" & call algebraic (D. called it "mechanical")

These lead  
to calculus &  
analysis

{ • all other curves (e.g. the graphs of  $\sin x, \cos x, \tan x, e^x$ ) are to be called transcendental (D. = "geometric")

Finally, we should observe an important feature of Descartes' algebraization of geometry: the new symbolic formalism.

Algebra existed before Descartes, but Descartes gave it shape, its modern shape. He introduced many current conventions:

- using lower case letters  $a, b, c, \dots$   
to represent known quantities
- using the end of the alphabet,  $x, y, z, \dots$ ,  
to represent unknown quantities,
- exponential notation,  $x^3, x^4$  etc.