ch 4

(2) x odd $\Rightarrow x^3$ odd

$x = 1 \in \mathbb{Z}_2 \Rightarrow x^3 = 1 \in \mathbb{Z}_2$

(4) x, y odd $\Rightarrow xy$ odd

i.e.

$x, y = 1 \in \mathbb{Z}_2 \Rightarrow xy = 1 \in \mathbb{Z}_2$

(6) $a, b, c \in \mathbb{Z}$. $a|b$ & $a|c \Rightarrow a|(b+c)$

(8) $a \in \mathbb{Z}$, $5|2a \Rightarrow 5|a$

pf: $5|2a \Rightarrow 2a = 5k$

$\Rightarrow 5k$ is even

$\Rightarrow k$ is even (else, if $k = 2r+1$,

$5k = 5(2r+1) = 10r+5$

$= 2(5r+2)+1$

is odd)

$\Rightarrow k = 2l \Rightarrow \cancel{2}a = 5k = 5(2l) = \cancel{2}(5l)$

$a = 5l$

$\Rightarrow 5|a$. QED

(14) $n \in \mathbb{Z} \Rightarrow 5n^2+3n+7$ is odd

(i) case 1, $n=2k$: $5n^2+3n+7 = 5(2k)^2+3(2k)+7$

$$= 2(5\cdot 2k^2+3k+2)+1$$

odd

(ii) case 2, $n=2k+1$;

$$5n^2+3n+7 = 5(2k+1)^2+3(2k+1)+7$$

$$= 5(4k^2+4k+1)+3(2k+1)+7$$

$$= \underbrace{20k^2+26k}_{\text{even \#}} + \underbrace{5+3+7}_{15=2\cdot 7+1}$$

$$= 2(10k^2+13k+7)+1$$

odd.

(18) $0<x<y \Rightarrow 0<x^2<y^2$

pf: $0<x<y \Rightarrow x^2<xy<y^2 \Rightarrow x^2<y^2$

& $0<x \Rightarrow 0 = 0\cdot x$

$< x\cdot x$

$= x^2$

(A.3.2) of $\mathbb{R}$ or Thm. 3.12 (ii) "Numbers"

(20) $a \in \mathbb{Z}$ & $a^2 | a \Rightarrow a \in \{-1, 0, 1\}$

pf: $a^2 | a \Rightarrow a = a^2 k$

$\Rightarrow 1 = ak$

(i) $a \neq 0$

$\Rightarrow a = k = \pm 1$

(ii) $a = 0 \Rightarrow 0 = a \overset{\text{def}}{=} a^2 k = 0^2 k \overset{\text{L.5.4 R}}{=} 0k = 0.$

(26) every odd integer is the diff. of 2 squares,

e.g. $7 = 4^2 - 3^2$

$(= 16 - 9)$

pf: actually, notice: $7 = 4^2 - 3^2 = (3+1)^2 - 3^2$

$= \cancel{3^2} + 2 \cdot 3 + 1 - \cancel{3^2}$

$= 2 \cdot 3 + 1$ ✓