

Quiz 6

1. Find the volume of region bounded by the upper half of the sphere $x^2 + y^2 + z^2 = 9$ and the plane $z = 1$. [Hint: Use cylindrical coordinates.]

Recall that in Quiz 5 we set up the integral in xyz -coordinates and got

$$V = \int_{-\sqrt{8}}^{\sqrt{8}} \int_{-\sqrt{8-x^2}}^{\sqrt{8-x^2}} \int_1^{\sqrt{9-x^2-y^2}} 1 \, dz \, dy \, dx$$

which was basically impossible to evaluate exactly. We'll get an exact answer, however, by using cylindrical coordinates. Namely, we realize $0 \leq \theta \leq 2\pi$, $1 \leq z \leq 3$ and $0 \leq r \leq \sqrt{9-z^2}$, since $r^2 + z^2 = (x^2 + y^2) + z^2 = 9$:

$$\begin{aligned} V &= \int_0^{2\pi} \int_1^3 \int_0^{\sqrt{9-z^2}} r \, dr \, dz \, d\theta \\ &= 2\pi \int_1^3 \left[\frac{1}{2} r^2 \right]_0^{\sqrt{9-z^2}} dz \\ &= \pi \int_1^3 (9 - z^2) dz \\ &= \pi \left(9z - \frac{1}{3} z^3 \right) \Big|_1^3 \\ &= \pi \left(27 - 9 - 9 + \frac{1}{3} \right) \\ &= \boxed{\frac{28\pi}{3}} \end{aligned}$$