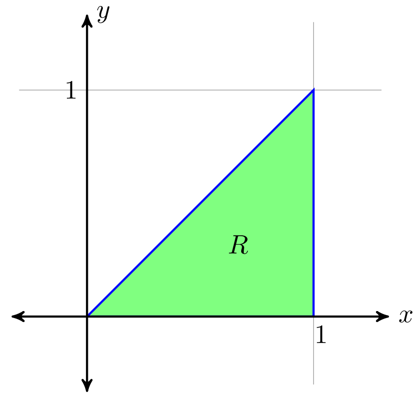
Quiz 5

1. Evaluate the integral $\int_0^1 \int_y^1 e^{x^2}\,dx\,dy$.

   First, note that as things stand we can't compute the integral, because e^{x^2} has no anti-derivative. We can perhaps hope to resolve this problem by switching the order of integration, but to do that we need to know what the new bounds would be. To find out, let's draw a picture of the region R over which we're integrating. Since x goes between $x = y$ and $x = 1$ and y goes between 0 and 1, R is the triangle



   Thus, if we integrate with respect to y first, y will go between $y = 0$ and $y = x$, then x will go between 0 and 1, so

$$\int_0^1 \int_y^1 e^{x^2}\,dx\,dy = \int_0^1 \int_0^x e^{x^2}\,dy\,dx = \int_0^1 \left[ye^{x^2}\right]_{y=0}^{x} dx = \int_0^1 xe^{x^2}\,dx = \frac{1}{2}\int_0^1 e^u\,du = \boxed{\frac{e-1}{2}}$$

   where we used the substitution $u = x^2$, $du = 2x\,dx$, or $\frac{1}{2}du = x\,dx$.

2. Find the volume of the region under the graph of $f(x, y) = xy$ and above the square $0 \le x \le 2$, $0 \le y \le 2$.

   This is a simple iterated integral:

$$V = \int_0^2 \int_0^2 xy\,dx\,dy = \int_0^2 \left[\frac{1}{2}x^2y\right]_{x=0}^{2} dy = \int_0^2 2y\,dy = y\Big|_{y=0}^{2} = \boxed{4}$$

3. Set up the integral, in cartesian coordinates, needed to find the volume of region bounded by the upper half of the sphere $x^2 + y^2 + z^2 = 9$ and the plane $z = 1$.

   I'm just going to set up the integral in xyz-coordinates without evaluating it, because that would be needlessly painful. We will evaluate this integral using cylindrical coordinates later.

$$\boxed{V = \int_{-\sqrt{8}}^{\sqrt{8}} \int_{-\sqrt{8-x^2}}^{\sqrt{8-x^2}} \int_{1}^{\sqrt{9-x^2-y^2}} 1 \, dz \, dy \, dx}$$