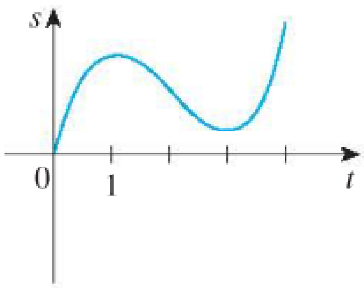
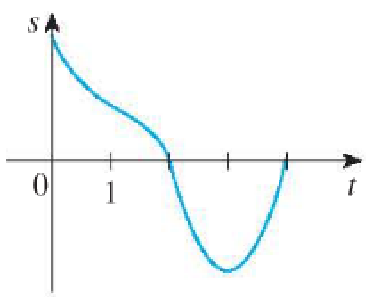
Turn in the following problems:

1. Graphs of the *position* functions of two particles are shown, where t is measured in seconds. When is each particle speeding up? When is it slowing down? Explain.

(a)


(b)


2. A HALO jumper with a mass of m parachutes from a gas balloon at time $t = 0$. Under certain assumptions, the distance, $s(t)$, he has fallen in time t is given by

$$s(t) = \frac{m^2 g}{k^2}\left(\frac{kt}{m} + e^{-kt/m} - 1\right) \text{ for some positive constant } k.$$

   (a) Find $s'(0)$ and $s''(0)$ and interpret in terms of the HALO jumper.
   (b) Relate the units of $s'(t)$ and $s''(t)$ to the units of t and $s(t)$.

3. Find the linear approximation of the function $g(x) = \sqrt[3]{1+x}$ at $a = 0$ and use it to approximate the numbers $\sqrt[3]{0.95}$ and $\sqrt[3]{1.1}$. Illustrate by graphing g and the tangent line.

4. The table shows the population of Nepal (in millions) of the given year. Use a linear approximation to estimate the population at midyear in 1992. Use another linear approximation to predict the population in 2020.

t	$N(t)$
1985	16.72
1990	18.75
1995	21.39
2000	23.74
2005	25.64
2010	27.02
2015	28.65

5. Use logarithmic differentiation to find the derivative of the function.

   (a) $y = \sqrt{x}e^{x^2}(x^2+1)^{10}$

   (b) $y = x^{\cos(x)}$

   (c) $\ln(x) + \ln(y^2) = 3$

6. The length of the day in Boulder (Latitude 40 N) can be modeled approximately by

$$l(t) = -3\cos\left(\frac{2\pi}{365}(t+10)\right) + 12$$

where l is given in hours and t is the day of the year.

   (a) Evaluate $l(355)$; fully interpret the result in the context of this problem, including units.

   (b) Evaluate $l'(265)$; fully interpret the result in the context of this problem, including units.

   (c) Calculate when $l'(t)$ is largest. Explain.

These problems will not be collected, but you might need the solutions during the semester:

7. A farmer plants lavender to attract a greater number of bees to the area. There are 100 bees in the area naturally, and for each acre of lavender planted, an additional 20 bees are found in the area.

   (a) Draw a graph of the total number, $B(a)$, of bees as a function of a, the number of acres of lavender planted.

   (b) Explain, both geometrically and algebraically, the shape of the graph of:

      i. The marginal rate of increase of the number of bees with acres of lavender, $B'(a)$.

      ii. The average number of bees per acre of lavender, $B(a)/a$.

8. Find a formula for $f^{(n)}(x)$ of $f(x) = \ln(x-1)$.

9. The U.S. gross domestic product can be modeled by

$$P(t) = 4.351e^{0.0368t}$$

   where P is given in billions of dollars and t is years since 1790.

   (a) Find $P(244)$; fully interpret the result in the context of this problem, including units.

   (b) When was the GDP one trillion dollars?

   (c) How many years does it take for the GDP to double?

   (d) What is $P'(224)$? Again, fully interpret (including units).

10. (a) On what interval is $f(x) = x\ln(x)$ decreasing?

    (b) On what interval is f concave upward?

11. Let $f(x) = \log_a(3x^2 - 2)$. For what value of a is $f'(1) = 3$?

12. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$.

    (a) What is the maximum height reached by the ball?

    (b) What is the velocity of the ball when it is 96 ft above the ground on its way up? On its way down?

13. The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the function

$$n = f(t) = \frac{a}{1 + be^{-0.7t}}$$

where t is measured in hours. At time $t = 0$ the population is 20 cells and is increasing at a rate of 12 cells/hour. Find the values of a and b. According to this model, what happens to the yeast population in the long run?

14. The cost function for production of a commodity is

$$C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$$

(a) Find and interpret $C'(100)$.

(b) Compare $C'(100)$ with the cost of producing the 101st item.

15. Explain, in terms of linear approximations or differentials, why the approximation is reasonable.

$$(1.01)^6 \approx 1.06$$

16. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemispherical dome with diameter 50 m.