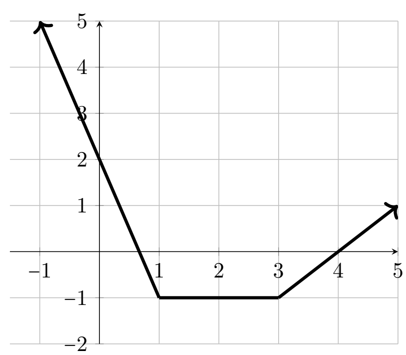
Turn in the following problems:

1. Use the function $f(x) = 4x^3e^x$ to answer the following problems:
   (a) On which interval(s) is the function $f(x)$ increasing?
   (b) On which interval(s) is the function $f(x)$ concave upward?

2. (a) If $F(x) = f(x)g(x)$, where f and g have derivatives of all orders, show that
$$F'' = f''g + 2f'g' + fg''.$$
   (b) Find similar formulas for F''' and $F^{(4)}$.
   (c) Guess a formula for $F^{(n)}$.

3. A table of values for the functions $f(x)$ and $f'(x)$ and a graph of the piecewise linear function $g(x)$ are shown below.

x	$f(x)$	$f'(x)$
-1	11	-7
0	2	-2
1	-2	5
2	9	3
3	0	4
4	1	2



$g(x)$

   (a) Given $h(x) = f(x)g(x)$, find $h'(1)$.
   (b) Given $p(x) = \dfrac{f(x)}{g(x)}$, find $p'(2)$.
   (c) Given $q(x) = \dfrac{g(x)}{f(x)}$, find $q'(2)$.
   (d) Given $q(x) = \dfrac{f(x)}{g(x)}$, find $q'(3)$.
   (e) $l(x) = \dfrac{g(x)}{\sqrt{x}}$, find $l'(4)$.

4. Prove that $\dfrac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$.

5. Consider the following mathematical statements. Determine if the statements are always true, sometimes true, or never true.

   - If the statement is always true, then give a brief explanation of why it is true.
   - If the statement is sometimes true, then give two specific examples: one where the statement is true and one where the statement is not true. Be sure to indicate which example is which.
   - If the statement is never true, then give a specific counterexample (an example where the statement is not true).

   An example must include either a graph or a specific function.

   (a) If $\dfrac{f(x)}{g(x)}$ is defined but not differentiable at $x = 1$, then either $f(x)$ or $g(x)$ is not differentiable at $x = 1$.

   (b) If f and g are two functions whose second derivatives are defined, then
   $$(f \cdot g)'' = f \cdot g'' + f'' \cdot g.$$

   (c) $(f(x) \cdot g(x))' = f'(x) \cdot g'(x)$.

   (d) If $f(x) = \dfrac{(1-x)}{e^{-x}}$, then $f(x)$ cannot be differentiated using the product rule.

   (e) If $h(x) = \dfrac{x^2}{e^{-x}}$, then $h'(x) = \dfrac{2x}{-e^{-x}}$.

   In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the "if" part of the statement is true, but the "then" part of the statement is false.

6. A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.

   (a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$?

   (b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

These problems will not be collected, but you might need the solutions during the semester:

7. If f is a differentiable function, find an expression for the derivative fo the following function:

$$y = \frac{1 + xf(x)}{\sqrt{x}}$$

8. A ladder 10 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \dfrac{\pi}{3}$?

9. Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

   (a) $\dfrac{d^{99}}{dx^{99}}(\sin(x))$

   (b) $\dfrac{d^{35}}{dx^{35}}(x \sin(x))$

Optional Challenge Problems

How many tangent lines to the curve $y = x/(x+1)$ pass through the point $(1,2)$? At which points do these tangent lines touch the curve?