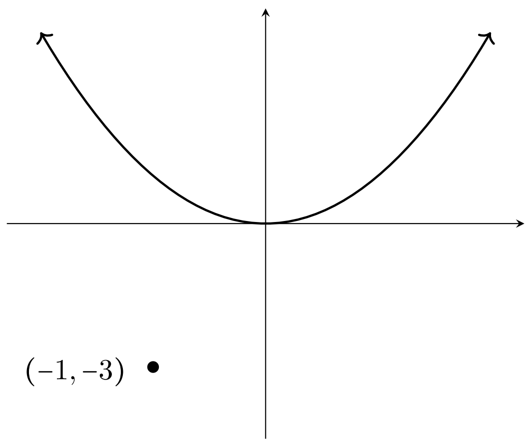
Turn in the following problems:

1. Find equations for the two distinct tangent lines to the curve $y = x^2$ through the point $(-1, -3)$.

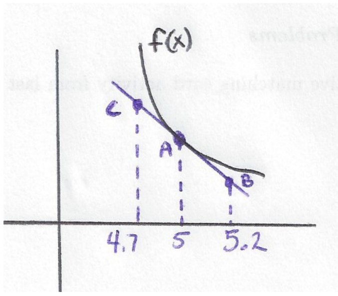


2. Sketch the graph of a function that satisfies all of the given conditions.

- $f'(2) = f'(-2) = 0$
- $f'(x) < 0$ if $|x| < 2$
- $f'(x) > 0$ if $2 < |x| < 3$
- $f'(x) = -1$ if $|x| > 3$
- $f''(x) < 0$ if $-2 < x < 0$
- inflection point $(0, -2)$

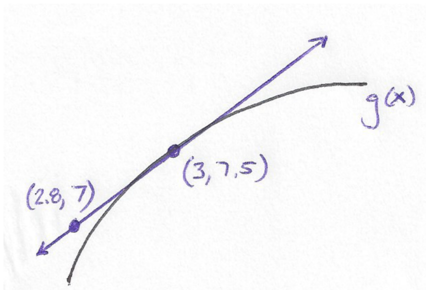
It can be helpful from time to time to sketch a graph or picture to strengthen our understanding of concepts and ideas. Use the following sketches of functions and tangent lines to answer problems below.

3. A sketch of the function $f(x)$ is given below:



If $f(5) = 12$ and $f'(5) = -2$, then find the coordinates of A, B, and C.

4. A sketch of the function $g(x)$ is given below:



If possible, find each of the following. Write "Not enough information" where appropriate.

(a) $g(3) =$

(b) $g(2.8) =$

(c) $g^{-1}(7) =$

(d) $g'(3) =$

(e) $g'(2.8) =$

5. The equation $y'' + y' - 2y = x^2$ is called a **differential equation** because it involves an unknown function y and its derivatives y' and y''. Find constants A, B, and C such that the function $y = Ax^2 + Bx + C$ satisfies this equation. (Differential equations will be studied in detail in Calculus 2).

6. Consider the following mathematical statements. Determine if the statements are always true, sometimes true, or never true.

   - If the statement is always true, then give a brief explanation of why it is true.
   - If the statement is sometimes true, then give two specific examples: one where the statement is true and one where the statement is not true. Be sure to indicate which example is which.
   - If the statement is never true, then give a specific counterexample (an example where the statement is not true).

   An example must include either a graph or a specific function.

   (a) If $f''(x) > 0$ on the interval (a, b), then $f'(x) < 0$ on the interval (a, b).

   (b) If $f(x)$ is a polynomial, then it is differentiable for all x.

   (c) The tangent line to $f(x)$ at $x = a$ will intersect the graph of $f(x)$ at one point.

   In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the "if" part of the statement is true, but the "then" part of the statement is false.

These problems will not be collected, but you might need the solutions during the semester:

7. Suppose $f'(x) = xe^{-x^2}$
   (a) On what interval is f increasing? On what interval is f decreasing?
   (b) Does f have a maximum value? Minimum value?

8. The president announces that the national deficit is increasing, but at a decreasing rate. Interpret this statement in terms of a function and its derivatives.

9. On what interval is the function $f(x) = x^3 - 4x^2 + 5x$ concave upward?

10. Suppose the curve $y = x^4 + ax^3 + bx^2 + cx + d$ has a tangent line when $x = 0$ with equation $y = 2x + 1$ and a tangent line when $x = 1$ with equation $y = 2 - 3x$. Find the values of a, b, c, and d.

Optional Challenge Problems

Find a possible formula for each function in the derivative matching card activity from last week (see the "Activities" page of the course website).