Turn in the following problems:

1. Given $f(x) = x^{-2}$. Evaluate your classmate's work using the limit definition of the derivative to find $f'(a)$. Their work is below:

   (i) $\displaystyle \lim_{h\to 0} \frac{(a+h)^{-2} - a^{-2}}{h}$

   (ii) $\displaystyle = \lim_{h\to 0} \frac{\frac{1}{(a+h)^2} - \frac{1}{a^2}}{h}$

   (iii) $\displaystyle = \lim_{h\to 0} \frac{\frac{a^2-(a+h)^2}{a^2(a+h)^2}}{h}$

   (iv) $\displaystyle = \lim_{h\to 0} \frac{\frac{a^2-a^2+ah+h^2}{a^2(a+h)^2}}{h}$

   (v) $\displaystyle = \lim_{h\to 0} \frac{\frac{ah+h^2}{a^2(a+h)^2}}{h}$

   (vi) $\displaystyle = \lim_{h\to 0} \frac{a+h}{a^2(a+h)^2}$

   (vii) $\displaystyle = \frac{a}{a^2(a)^2} = a^{-3}$

   If your classmate made any errors in their work, state which line(s) where the error(s) occurred (e.g., line (ii)), explain the error(s), and then correctly find $f'(a)$.
   If your classmate made no errors in their work, then write 'Excellent work, that was a challenging problem!'.

2. Sketch the graph of an example of a function f that satisfies all of the given conditions:

   - $\lim_{x\to 2} f(x) = \infty$
   - $\lim_{x\to -2^+} f(x) = \infty$
   - $\lim_{x\to -2^-} f(x) = -\infty$
   - $\lim_{x\to -\infty} f(x) = 0$
   - $\lim_{x\to \infty} f(x) = 0$
   - $f(0) = 0$

3. Sketch the graph of a function f for which $f(0) = 0, f'(0) = 3, f'(1) = 0$, and $f'(2) = -1$.

4. Let $h(t)$ be the height of the tide at the Bay of Fundy in meters that has changed since midnight, with t measured in hours. We restrict the domain of $h(t)$ to where it is a one-to-one function. Interpret the following in practical terms, giving units.

   (a) $h(7) = 2.75$
   (b) $h'(7) = 0.21$
   (c) $h^{-1}(-1.5) = 13.2$
   (d) $(h^{-1})'(-1.5) = -1.6$

5. Let $V(t)$ be the volume of water in a tank (in liters), at time t (in seconds).

   (a) What are the meaning and units of $\dfrac{dV}{dt}$?
   (b) The tank is full at time t_0, so that $V(t_0) > 0$. At some later time t_1 a drain is opened 20 cm above the bottom of the tank (which is taller than 20cm), emptying water from the side of the tank. Is $\dfrac{dV}{dt}$ positive, negative, or zero at the following times:
      i. at time t with $t_0 < t < t_1$?
      ii. after the drain has been opened at t_1, but before the water has dropped to 20 cm above the bottom of the tank?
      iii. after the water drops to the drain hole 20 cm above the bottom of the tank?

6. The **derivative of a function** f **at a number** a denoted by $f'(a)$, is

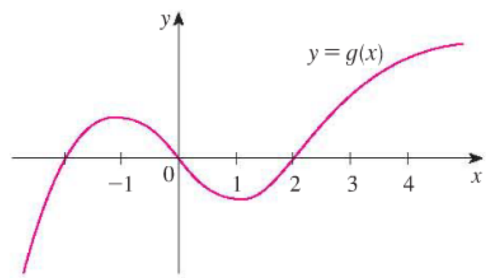
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

   if this limit exists.

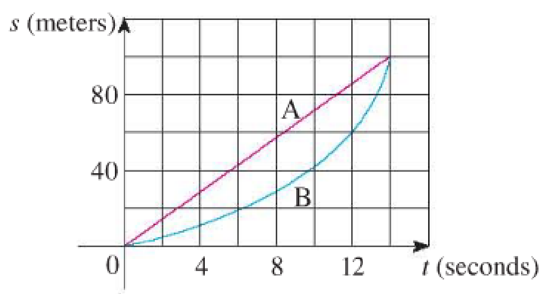
   Draw a visual representation for this definition using the function $f(x) = x^2$. Then explain how your illustration represents the definition of the derivative a function at a number a.

These problems will not be collected, but you might need the solutions during the semester:

1. For the function g whose graph is given, arrange the following numbers in increasing order and explain your reasoning:
   - 0
   - $g'(-2)$
   - $g'(0)$
   - $g'(2)$
   - $g'(4)$



2. Shown are graphs of the position functions of two runners, A and B, who run a 100-m race and finish in a tie.

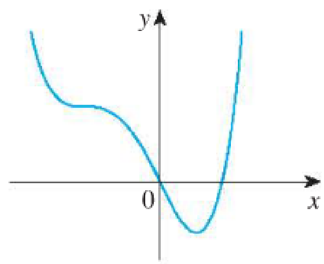


   (a) Describe and compare how the runners run the race.
   (b) At what time is the distance between the runners the greatest?
   (c) At what time do they have the same velocity?

3. Use the limit definition of the derivative to find $f'(1)$ given the following function:

$$f(x) = \frac{1}{1+x}$$

4. Trace or copy the graph of the given function f. (Assume that the axes have equal scales.) Then use the method outlined in Example 1 on page 146 of your text to sketch the graph of f' below it.



5. Let $P(t)$ be the percentage of Americans under the age of 18 at time t. The table gives values of this function in census years from 1950 to 2000.

t	$P(t)$	t	$P(t)$
1950	31.1	1980	28.0
1960	35.7	1990	25.7
1970	34.0	2000	25.7

(a) What is the meaning of $P'(t)$? What are its units?

(b) Construct a table of estimated values for $P'(t)$.

(c) Graph P and P'.

(d) How would it be possible to get more accurate values for $P'(t)$?