Turn in the following problems:

1. The point $P(0.5, 0)$ lies on the curve $y = \cos(\pi x)$.

   (a) If Q is the point $(x, \cos(\pi x))$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:

      i. 0
      ii. 0.4
      iii. 0.49
      iv. 0.499
      v. 1
      vi. 0.6
      vii. 0.51
      viii. 0.501

   (b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 0)$.

   (c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(0.5, 0)$.

   (d) Sketch the curve, two of the secant lines, and the tangent line.

2. Consider the following mathematical statements. Determine if the statements are always true, sometimes true, or never true.

   - If the statement is always true, then give a brief explanation of why it is true.
   - If the statement is sometimes true, then give two specific examples: one where the statement is true and one where the statement is not true. Be sure to indicate which example is which.
   - If the statement is never true, then give a specific counterexample (an example where the statement is not true).

   An example must include either a graph or a specific function.

   (a) If $\lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ does not exist.

   (b) If $\lim_{x\to a^+} g(x) = 2$ and $\lim_{x\to a^-} g(x) = -2$, then $\lim_{x\to a} (g(x))^2$ exists.

   (c) If $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} (f(x) + g(x))$ exists, then $\lim_{x\to a} g(x)$ exists.

   (d) $\lim_{x\to 1} \dfrac{|x-1|}{x-1} = 1$

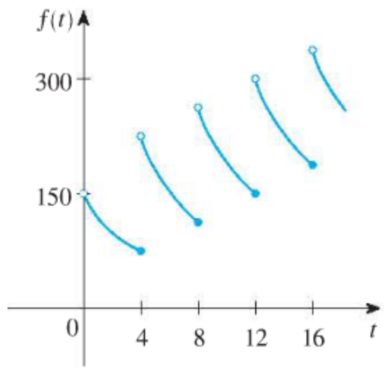
   (e) If f is a function, then $\lim_{x\to a} f(x) = f(a)$

   In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the "if" part of the statement is true, but the "then" part of the statement is false.

3. A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find

$$\lim_{t\to 12^-} f(t) \text{ and } \lim_{t\to 12^+} f(t)$$

and explain the significance of these one-sided limits.



4. Sketch the graph of an example of a function f that satisfies all of the given conditions.

- $\lim_{x\to 1} f(x) = 2$
- $\lim_{x\to 3^-} f(x) = -4$
- $\lim_{x\to 3^+} f(x) = 4$
- $f(1) = 0$
- $f(3) = 4$

5. Sketch the graph of the function and use it to determine the value of a for which $\lim_{x\to a} f(x)$ does not exists.

$$f(x) = \begin{cases} 1+\sin(x) & \text{if } x < 0 \\ \cos(x) & \text{if } 0 \le x \le \pi \\ \sin(x) & \text{if } x > \pi \end{cases}$$

6. (a) What is wrong with the following equation?

$$\frac{x^4 - 8x^2 + 16}{x^2 - 4} = x^2 - 4$$

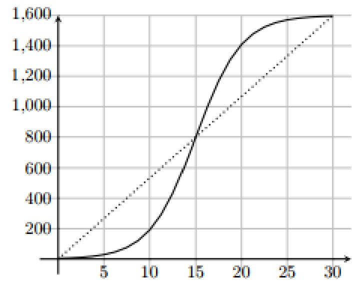
(b) In view of part (a), explain why the equation

$$\lim_{x \to 2} \frac{x^4 - 8x^2 + 16}{x^2 - 4} = \lim_{x \to 2} (x^2 - 4)$$

is correct.

These problems will not be collected, but you might need the solutions during the semester:

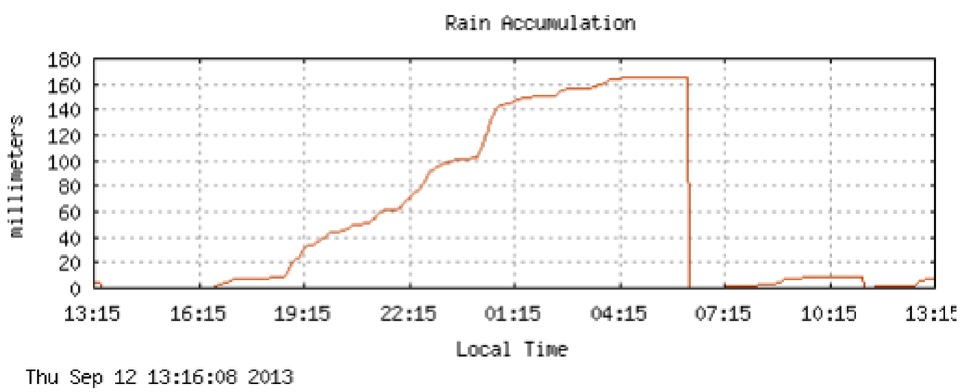
1. The solid curve in the graph below gives position s of a car along a straight roadway (measured in meters), as a function of time t (measured in seconds).



   (a) Find the slope of the dotted line in the graph above. Explain (including units), what this slope represents.

   (b) Estimate the instantaneous velocity at $t = 15$. Include units. Draw and label the line you used to estimate this.

2. Below is a plot of the rainfall accumulation from the 2013 Boulder flood taken from the Foothills Lab Weather Station. The rainfall is measured in millimeters.



   (a) Use the graph to estimate the average rainfall rate between 4:15 pm (marked as 16:15 on the graph) and 4:15 am the next morning (marked as 04:15 on the graph). Show all work and include units. Draw the line that you are finding the slope of.

   (b) When is it raining hardest? Explain how you know.

   (c) Estimate the rainfall rate at 22:15 (include units). Draw the line that you are finding the slope of.

   (d) What does the graph indicate is happening to the rainfall during the hour after 4:15 am?

   (e) Explain the precipitous drop between 04:15 and 07:15.

Optional Challenge Problem

Complete 5g from Project 1