Turn in the following problems:

1. Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways: (a) by using Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

$$g(x) = \int_0^x (1 + \sqrt{t})\, dt$$

2. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$y = \int_{\sin(x)}^{\cos(x)} (1 + v^2)^{10}\, dv$$

3. On what interval is the curve

$$y = \int_0^x \frac{t^2}{t^2 + t + 2}\, dt$$

concave downward?

4. Let

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x \le 1 \\ 2 - x & \text{if } 1 < x \le 2 \\ 0 & \text{if} x > 2 \end{cases}$$

and

$$g(x) = \int_0^x f(t)\, dt$$

(a) Find an expression for $g(x)$ similar to the one for $f(x)$.
(b) Sketch the graphs of f and g.
(c) Where is f differentiable? Where is g differentiable?

5. If f is continuous and $\int_0^9 f(x)\, dx = 4$, find $\int_0^3 x f(x^2)\, dx$.

6. Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

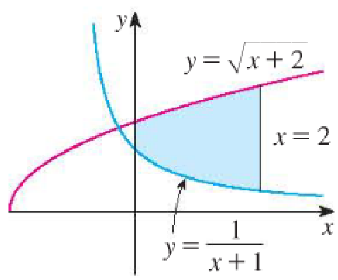
$$4x + y^2 = 12, \; x = y$$

These problems will not be collected, but you might need the solutions during the semester:

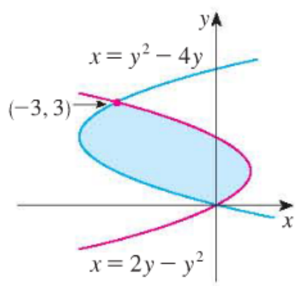
7. Evaluate the indefinite integral

$$\int \frac{dx}{5-3x}$$

8. Find the area of the shaded region.



(a)



(b)

9. If f is continuous on $\mathbb{R}$, prove that

$$\int_a^b f(x+c)\,dx = \int_{a+c}^{b+c} f(x)\,dx$$

For the case where $f(x) \geq 0$, draw a diagram to interpret this equation geometrically as an equality of areas.

10. Evaluate exactly the following integrals

(a) $\displaystyle\int_0^{\sqrt[4]{3}} \frac{x}{1+x^4}\,dx$

(b) $\displaystyle\int_0^{\pi/4} \sec^6(x)\tan(x)\,dx$