Turn in the following problems:

1. Consider the following mathematical statements. Determine if the statements are always true, sometimes true, or never true.

   - If the statement is always true, then give a brief explanation of why it is true.
   - If the statement is sometimes true, then give two specific examples: one where the statement is true and one where the statement is not true. Be sure to indicate which example is which.
   - If the statement is never true, then give a specific counterexample (an example where the statement is not true).

   An example must include either a graph or a specific function.

   (a) For any function f, if $f''(0) = 0$, there is an inflection point at $x = 0$.

   (b) If f is a function and $f'(p) = 0$, then f has a local minimum or maximum at $x = p$.

   (c) A local minimum of a function f occurs at a critical point of f.

   (d) f' is continuous and f has no critical points, then f is everywhere increasing or everywhere decreasing.

   In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the "if" part of the statement is true, but the "then" part of the statement is false.

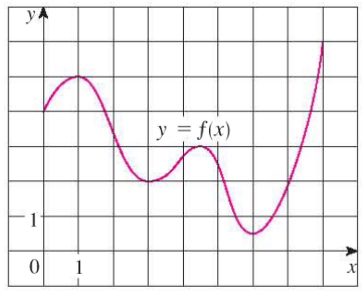
2. Sketch the graph of a function, f, that satisfies the following properties:

   - $x = -2$ is a vertical asymptote
   - $\lim_{x \to -\infty} f(x) = 3$
   - $f'(0) = 0$ and $f'(5) = 0$
   - $f''(5) = 0$
   - $f''(2)$ is undefined
   - $f'(x) > 0$ on $(0, 2)$
   - $f'(x) < 0$ on $(-\infty, -2) \cup (-2, 0) \cup (2, 5) \cup (5, \infty)$
   - $f''(x) > 0$ on $(-2, 2) \cup (2, 5)$
   - $f''(x) < 0$ on $(-\infty, -2) \cup (5, \infty)$

3. (a) Sketch the graph of a function that has two local maxima, one local minimum, and no absolute minimum.

   (b) Sketch the graph of a function that has three local minima, two local maxima, and seven critical numbers.

4. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval $[0, 8]$.



5. Use the following function to answer the problems below.

$$f(x) = \frac{x^2}{(x-2)^2}$$

   (a) Find the vertical and horizontal asymptotes.
   (b) Find the intervals of increase or decrease.
   (c) Find the local maximum and minimum values.
   (d) Find the intervals of concavity and the inflection points.
   (e) Use the information from parts (a)-(d) to sketch the graph of f.

6. A cubic function is a polynomial of degree 3; that is, it has the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

   (a) Show that a cubic function can have two, one, or no critical number(s). Give examples and sketches to illustrate the three possibilities.
   (b) How many local extreme values can a cubic function have?

These problems will not be collected, but you might need the solutions during the semester:

7. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin(\theta) + \cos(\theta)}$$

where μ is a positive constant called the *coefficient of friction* and where $0 \le \theta \le \pi/2$. Show that F is minimized when $\tan(\theta) = \mu$.

8. (a) Find the critical numbers of $f(x) = x^4(x-1)^3$.
   (b) What does the Second Derivative Test tell you about the behavior of f at these critical numbers?
   (c) What does the First Derivative Test tell you?

9. At 2:00 PM a car's speedometer reads 30 mi/h. At 2:10 PM it reads 50 mi/h. Show that at some time between 2:00 and 2:10 the acceleration is exactly 120 mi/h^2.

10. For what values of c does the polynomial $P(x) = x^4 + cx^3 + x^2$ have two inflection points? One inflection point? None? Illustrate by graphing P for several values c. How does the graph change as c decreases?

11. Find the absolute extrema of the function $f(x) = xe^{-x^2/18}$ on the interval $[-2, 4]$.