Turn in the following problems:

1. Explain how we know there exists a tangent line to the curve $x^2 + 4y^2 = 8$ that passes through the point $(-4, 0)$. (Hint: It may be worth recalling the Intermediate Value Theorem)

2. Water is leaking out of an inverted conical tank at a rate of 1.5 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 10 cm and the diameter at the top is 6 cm. If the water level is rising at a rate of 1.0 cm/min when the height of the water is 2 cm, find the rate at which water is being pumped into the tank.
   (a) What quantities are given in the problem?
   (b) What is the unknown?
   (c) Draw a picture of the situation for any time t.
   (d) Write an equation that relates the quantities.
   (e) Finish solving the problem.

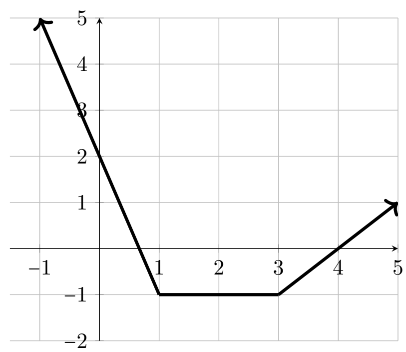
3. Explain what is wrong with the following statements:
   (a) Given $f(2) = 6, f'(2) = 3$, and $f^{-1}(3) = 4$, we have
   $$(f^{-1})'(2) = \frac{1}{f^{-1}(f'(2))} = \frac{1}{f^{-1}(3)} = \frac{1}{4}$$
   (b) If $\frac{dy}{dx} = \frac{-x}{y}$ gives the slope of the circle $x^2 + y^2 = 10$ at every point in the xy-plane except where $y = 0$.

4. Compute the derivatives for the following functions. Be sure to show all your work.
   (a) $C(q) = \sec\left(eq^2 + \pi\right)$
   (b) $s(t) = \dfrac{3}{\sqrt[5]{t^2 - 3\cos(t) - 1}}$
   (c) $h(x) = (3x^2 - x) \cdot \dfrac{(x-2)}{(2+x)}$

5. If $xy + e^y = e$, find the value of y'' at the point where $x = 0$.

6. A table of the functions $f(x)$ and $f'(x)$ and a graph of the piecewise linear function $f(x)$ are shown below.

x	$f(x)$	$f'(x)$
-1	11	-7
0	2	-2
1	-2	5
2	9	3
3	0	4
4	1	2



$y = g(x)$

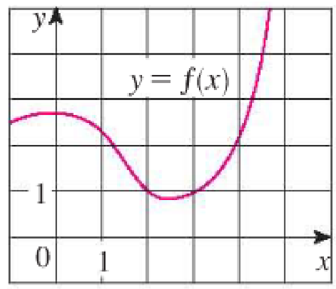
(a) Given $h(x) = f(g(x))$, find $h'(0)$.

(b) Given $p(x) = (g(x))^3$, find $p'(2)$.

(c) Given $k(x) = g(f(x))^{2/3}$, find $k'(4)$.

These problems will not be collected, but you are expected to solve. You might need the solutions during the semester:

7. If f is the function whose graph is shown, let $h(x) = f(f(x))$ and $g(x) = f(x^2)$. Use the graph of f to estimate the value of each derivative.
   (a) $h'(2)$
   (b) $g'(2)$



8. Under certain circumstances a rumor spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}}$$

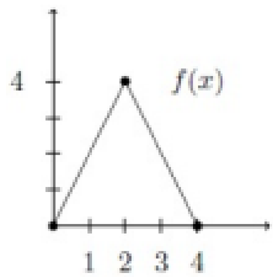
   where $p(t)$ is the proportion of the population that knows the rumor at time t and a and k are positive constants. (In Calculus 2 we will see that this is a reasonable equation for $p(t)$.)
   (a) Find $\lim_{t\to\infty} p(t)$
   (b) Find the rate of spread of the rumor.
   (c) Graph p for the case $a = 10$, $k = 0.5$ with t measured in hours. use the graph to estimate how long it will take for 80% of the population to hear the rumor.

9. Two curves are **orthogonal** if their tangent lines are perpendicular at each point of intersection. Show that the given families of curves are **orthogonal trajectories** of each other, that is, every curve in one family is orthogonal to every curve in the other family. Sketch both families of curves on the same axes.

$$y = ax^3 \text{ and } x^2 + 3y^2 = b$$

10. A particle moves along the curve $y = \sqrt{1 + x^3}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

11. The graph of $f(x)$ is shown and the table gives values of $g(x)$ and $g'(x)$.



x	0	1	2	3
$g(x)$	4	3	2	1
$g'(x)$	-1.1	-0.9	-1.2	-0.8

(The function $f(x)$ is piecewise linear)

(a) Given $h(x) = f(g(x))$, find $h'(1)$.

(b) Given $k(x) = g(f(x))$, find $k'(3)$.

(c) Given $l(x) = g(g(x))$, find $l'(2)$.

(d) Given $m(x) = \sqrt{f(x)}$, find $m'(1)$.

Optional Challenge Problems

Sketch the circles $x^2 + y^2 = 1$ and $(x-3)^2 + y^2 = 4$. There is a line with positive slope that is tangent to both circles. Determine the points at which this tangent line touches the circle.