

Turn in the following problems:

1. Use the function $f(x) = 4x^3e^x$ to answer the following problems:

- (a) On what interval(s) are the function $f(x)$ increasing?
 (b) On what interval(s) are the function $f(x)$ concave upward?

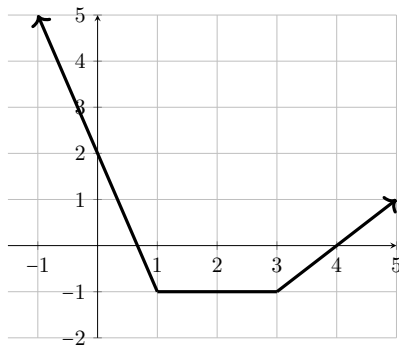
2. (a) If $F(x) = f(x)g(x)$, where f and g have derivatives of all orders, show that

$$F'' = f''g + 2f'g' + fg''.$$

- (b) Find similar formulas for F''' and $F^{(4)}$.
 (c) Guess a formula for $F^{(n)}$.

3. A table of the functions $f(x)$ and $f'(x)$ and a graph of the piecewise linear function $f(x)$ are shown below.

x	$f(x)$	$f'(x)$
-1	11	-7
0	2	-2
1	-2	5
2	9	3
3	0	4
4	1	2



$y = g(x)$

(a) Given $h(x) = f(x)g(x)$, find $h'(1)$.

(b) Given $p(x) = \frac{f(x)}{g(x)}$, find $p'(2)$.

(c) Given $q(x) = \frac{g(x)}{f(x)}$, find $q'(2)$.

(d) Given $k(x) = \frac{f(x)}{g(x)}$, find $k'(3)$.

(e) $l(x) = \frac{g(x)}{\sqrt{x}}$, find $l'(4)$.

4. Prove that $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$.
5. Consider the following mathematical statements. Determine if the statements are always true, sometimes true, or never true.
- (a) If $\frac{f(x)}{g(x)}$ is defined but not differentiable at $x = 1$, then either $f(x)$ or $g(x)$ is not differentiable at $x = 1$.
- (b) If f and g are two functions whose second derivatives are defined, then
- $$(f \cdot g)'' = f \cdot g'' + f'' \cdot g.$$
- (c) $(f(x) \cdot g(x))'$ and $f'(x) \cdot g'(x)$ are never equal.
- (d) If $f(x) = \frac{(1-x)}{e^{-x}}$, then $f(x)$ cannot be differentiated using the product rule.
- (e) If $h(x) = \frac{x^2}{e^{-x}}$, then $h'(x) = \frac{2x}{-e^{-x}}$.

In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the “if” part of the statement is true, but the “then” part of the statement is false.

With this in mind, now determine if the above statements are true or false. If the statement is true, give a brief explanation of why it is true. If the statement is false, give a counterexample. Be sure to explain why your counterexample shows the statement to be false.

6. A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.
- (a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$?
- (b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

These problems will not be collected, but you might need the solutions during the semester:

7. If f is a differentiable function, find an expression for the derivative for the following function:

$$y = \frac{1 + xf(x)}{\sqrt{x}}$$

8. A ladder 10 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \frac{\pi}{3}$?

9. Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

(a) $\frac{d^{99}}{dx^{99}}(\sin(x))$

(b) $\frac{d^{35}}{dx^{35}}(x \sin(x))$

Optional Challenge Problems

How many tangent lines to the curve $y = x/(x + 1)$ pass through the point $(1, 2)$? At which points do these tangent lines touch the curve?