

Turn in the following problems:

1. A 2018 Dodge Challenger SRT Demon accelerates from 0 to 88 ft/sec (60 mph) in 2.3 seconds. This qualified it as the fastest manufacturer timed acceleration for 2018.
 - (a) Assuming that acceleration is constant, graph the velocity from $t = 0$ to $t = 2.2$
 - (b) How far does the car travel during this time?

2. Find the most general antiderivative of the function. (Check you answer by differentiation.)

$$f(x) = \frac{2 + x^2}{1 + x^2}$$

3. Speedometer readings for a motorcycle at 12-second intervals are given in the table.

t (s)	0	12	24	36	48	60
v (ft/s)	30	28	25	22	24	27

- (a) Estimate the distance traveled by the motorcycle during this time period using the velocities at the beginning of the time intervals.
- (b) Give another estimate using the velocities at the end of the time periods.
- (c) Are your estimates in parts (a) and (b) upper and lower estimates? Explain.

4. Consider the following mathematical statements. Determine if the statements are always true, sometimes true, or never true.
- (a) A function $f(x)$ has at most one derivative.
 - (b) If $f(x)$ is a linear function with positive slope, then an antiderivative, $F(x)$, is a linear function.
 - (c) If $f(x)$ is positive over an interval, then an antiderivative $F(x)$ is positive over the interval.
 - (d) If $f(x)$ is increasing over an interval, then an antiderivative $F(x)$ is positive over the interval.

In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the “if” part of the statement is true, but the “then” part of the statement is false.

With this in mind, now determine if the above statements are true or false. If the statement is true, give a brief explanation of why it is true. If the statement is false, give a counterexample. Be sure to explain why your counterexample shows the statement to be false.

Use the following definition for area to answer Problems 5 and 6:

Definition: The **area** A of the region S that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x]$$

5. Use the definition of area given above to find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

$$f(x) = x^2 + \sqrt{1 + 2x}, \quad 4 \leq x \leq 7$$

6. (a) Use the definition of area given above to find an expression for the area under the curve $y = x^3$ from 0 to 1 as a limit.
- (b) The following formula for the sum of the cubes of the first n integers is proved in Appendix F of your textbook. Use it to evaluate the limit in part (a).

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

These problems will not be collected, but you might need the solutions during the semester:

7. Two balls are thrown upward from the edge of the cliff 432 ft above the ground. The first is thrown at a speed of 48 ft/s and the other is thrown a second later with a speed of 24 ft/s. Do the balls ever pass each other?
8. As the flu spreads through a population, the number of infected people, I , is expressed by a function of the number of susceptible people S , by

$$I = k \ln\left(\frac{S}{S_0}\right) - S + S_0 + I_0, \text{ for } k, S_0, I_0 > 0$$

- (a) Find the maximum number of infected people.
- (b) The constant k is a characteristic of the particular strain of the flu; the constants S_0 and I_0 are the values of S and I when the flu starts. Which of the following affects the maximum possible value of I ? Explain
- The particular strain of flu, but not how it starts.
 - How the disease starts, but not the particular strain of flu.
 - Both the particular strain of the flu and how it starts.
9. (a) Batman was driving the Batmobile at 90 mph (=132 ft/sec), when he sees a brick wall directly ahead. When the Batmobile is 400 ft from the wall, he slams on the brakes, decelerating at a constant rate of 22 ft/s². Does he stop before he hits the brick wall? If so, how many feet to spare? If not, what is his impact speed?
- (b) Now the Joker had been driving next to Batman, also at 90 mph. But the Joker did not hit his brakes as soon as Batman, continuing for 1 second longer than Batman before hitting his brakes, decelerating at a constant rate of 22 ft/s². How fast is he going when he hits the wall? (Don't worry about Joker - he jettisoned at the last instant, to fight for another day!)