Turn in the following problems:

1. Prove that

$$\lim_{x\to\infty} \frac{\ln(x)}{x^p} = 0$$

for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x.

2. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account is

$$v = \frac{mg}{c}(1 - e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant. (In Calculus 2 we will be able to deduce this equation from the assumption that the air resistance is proportional to the speed of the object; c is the proportionality constant.)

   (a) Calculate $\lim_{t\to\infty} v$. What is the meaning of this limit?

   (b) For fixed t, use l'Hôpital's Rule to calculate $\lim_{c\to 0^+} v$. What can you conclude about the velocity of a falling object in a vacuum?

3. The Extreme Value Theorem guarantees that only continuous functions have global maxima and global minima over every closed, bounded interval.

   (a) If $f(x) = x^2$, then f has an global maxima on the interval $(0,2)$.

   (b) If a differentiable function $f(x)$ has a global maximum on the interval $0 \le x \le 10$, then $f'(0) \le 0$.

   (c) The function $f(x) = (x-3)^2(x-4)$ has a global maximum at $x = 3$ over the interval $0 \le x \le 5$.

In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the "if" part of the statement is true, but the "then" part of the statement is false.
With this in mind, now determine if the above statements are true or false. If the statement is true, give a brief explanation of why it is true. If the statement is false, give a counterexample. Be sure to explain why your counterexample shows the statement to be false.

4. A rectangular storage container with an open top is to have a volume of 10 m^3. Then length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides cost \$6 per square meter. Find the cost of materials for the cheapest such container.

5. A cylindrical can without a top is made to contain a volume of 2000 cubic centimeters of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

6. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?

These problems will not be collected, but you might need the solutions during the semester:

7. A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

8. Find the limit. Use l'Hôpital's Rule if appropriate. If there is a more elementary method, consider using it. If l'Hôpital's Rule does not apply, explain why.

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$$

9. Use l'Hôpital's Rule to help find the asymptotes of f. Then use them, together with the information from f' and f'', to sketch the graph. Check your work with a graphing device or program.

$$f(x) = xe^{-x^2}$$

10. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account is

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where g is the acceleration due to gravity and c is a positive constant. (In Calculus 2 we will be able to deduce this equation from the assumption that the air resistance is proportional to the speed of the object; c is the proportionality constant.)

   (a) Calculate $\lim_{t\to\infty} v$. What is the meaning of this limit?

   (b) For fixed t, use l'Hôpital's Rule to calculate $\lim_{c\to 0^+} v$. What can you conclude about the velocity of a falling object in a vacuum?

11. If f' is continuous, $f(2) = 0$, and $f'(2) = 7$, evaluate

$$\lim_{x\to 0}\frac{f(2+3x)+f(2+5x)}{x}$$

Optional Challenge Problem

This problem will help improve your algebra skills.

Graph the function $f(x) = (x-2)^{1/3}x^{2/3}$. Include calculation of the first and second derivatives and full analysis of the increasing/decreasing behavior, local extrema, concavity and inflection points, and the end behavior.