Turn in the following problems:

1. Consider the following mathematical statements. Determine if the statements are always true, sometimes true, or never true.

   (a) $(x+2)^4 = x^4 + 16$

   (b) $\sqrt{x^4 + 8x^2 + 16} = x^2 + 4$

   (c) If $(x+2)(x-3) = 2$, then $x+2 = 2$ and $x-3 = 2$.

   (d) If f and g are both even functions, then $f+g$ is even.

   (e) If $x < y$, then $kx < ky$ where k is a constant.

   In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the "if" part of the statement is true, but the "then" part of the statement is false.
   With this in mind, now determine if the above statements are true or false. If the statement is true, give a brief explanation of why it is true. If the statement is false, give a counterexample. Be sure to explain why your counterexample shows the statement to be false.

2. Molly and Keitany ran the 7 mile (11.26 kilometers) Falmouth Road Race. Molly ran at a constant pace of 3:05 per kilometer. Keitany ran every one-kilometer interval in exactly three minutes and seven seconds. For example, Keitany ran the one-kilometer distance from 4.57 kilometers to 5.57 kilometers in three minutes and seven seconds, and the one-kilometer distance from 10.1 kilometers to 11.1 kilometers in three minutes and seven seconds. At the end of the race, Keitany finished ahead of Molly. Explain how. Include a graph to help support your reasoning.

Optional Challenge:
Research suggests that humans could perhaps run up to 40 mph. With this information, is the above race humanly possible?

Optional Neat Stuff:
If you ran a race at an average pace of 3:07 per kilometer, did you run any single kilometer in exactly 3:07?

To explore this further, visit: Math on the Run

3. An electricity company charges its customers a fixed base charge of \$6 a month, plus 10 cents per kilowatt-hour (kWh) for the first 500 kWh, 11 cents per kWh for the next 500 kWh, and 15 cents for all additional kWh. Express the monthly cost E as a function of the amount x of electricity used. Then graph the function E for $0 \leq x \leq 2000$. Explain how your graph represents your function E.

4. In September, Lee ran in the Run Rabbit Run 50 mile race in Steamboat. The race is an out-and-back course, meaning that you start and run out from the base of Steamboat Ski Resort up to Rabbit Ears, and then turn around and retrace your path to the finish. The table below provides the timed splits through the various aid stations along the course.

Time	**Aid State (distance traveled from start)**
0:00:00	Start (0 miles)
1:21:51	Mount Werner Aid Station (6.4 miles)
2:22:52	Long Lake Aid Station (13.2 miles)
3:34:36	Base Camp Trailhead Aid Station (18.4 miles)
4:05:34	Dumont Aid Station (22.3 miles)
4:45:25	Rabbit Ears (25 miles)
5:13:17	Dumont Aid Station (27.7 miles)
6:02:43	Base Camp Trailhead Aid Station (31.6 miles)
7:07:11	Long Lake Aid Station (36.8 miles)
8:41:37	Mount Werner Aid Station (43.6 miles)
9:29:13	Finish (50 miles)

   (a) Explain if the table represents a function. If it does not, provide evidence supporting that it is not a function. If it is a function, describe the relationship between the independent variable and dependent variable in this scenario.

   (b) Construct a graph that depicts the relationship between time and the total distance traveled in the race.

   (c) Construct a graph that depicts the relationship between time and displacement from start/finish (relative distance to start/finish).

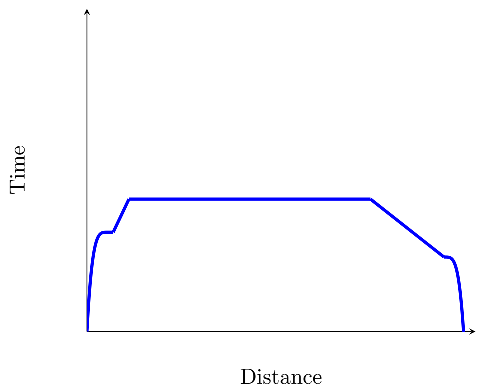
   (d) Construct a graph that depicts the relationship between time and Lee's speed running.

   (e) What can we infer about Lee's speed at time 4:05:34? Explain your reasoning.

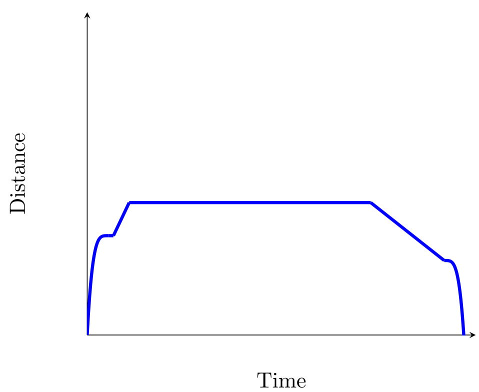
5. If you invest x dollars at 6% interest compounded annually, then the amount $A(x)$ of the investment after one year is $A(x) = 1.06x$. Find $A \circ A$, $A \circ A \circ A$, and $A \circ A \circ A \circ A$. What do these compositions represent? Find a formula for the composition of n copies of A.

6. In the morning Athena goes to the schools flagpole to raise Colorados state flag for the day. This is the first time she has gotten to raise the flag. She is so excited that she raises the flag at an increasing fast rate. However, the flagpole is higher than Athena anticipated, and she tires out and has to take a short break. After this break she raises the flag at a much slower steady rate until it reaches the top of the pole. At the end of the school day the child returns to take the flag down. Athena is nervous about how to lower the flag, and begins lowering the flag at a slow constant rate. As she becomes more comfortable she lowers the flag faster and faster until she gets the flag all the way down.

   For the above scenario choose ONE of the graphs below that provides a graphical representation of the relationship between time and the total distance traveled by the flag.



(a)



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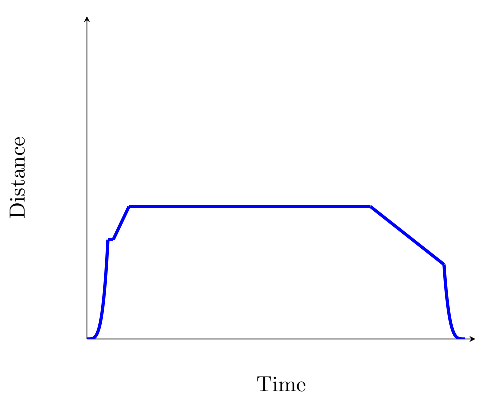

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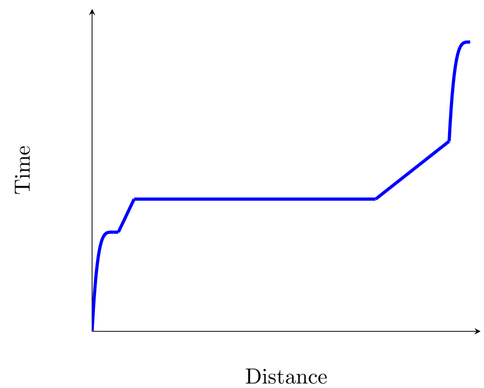
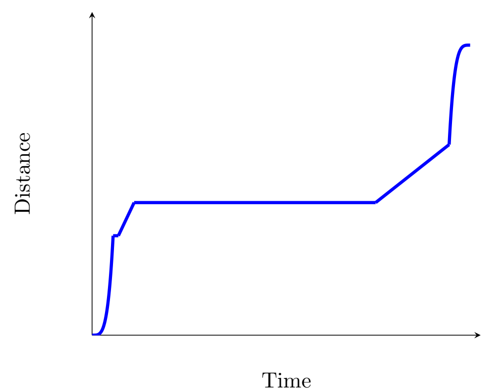
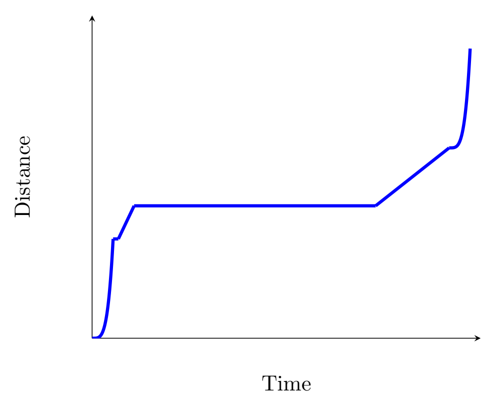


(d)



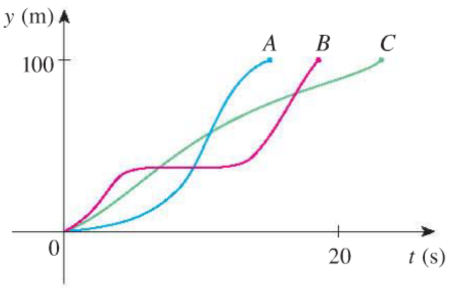
(e)



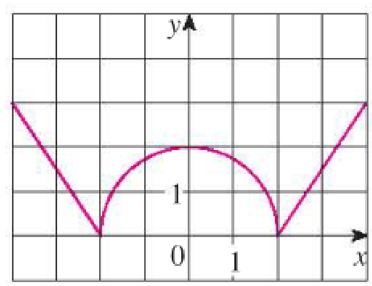
(f)

These problems will not be collected, but you are expected to solve. You might need the solutions during the semester:

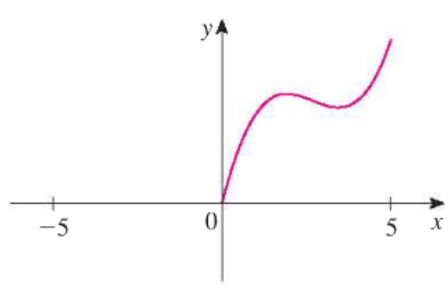
7. Three runners compete in a 100-meter race. The graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race. Who won the race? Did each runner finish the race?



8. Find the expression for the function whose graph is the given curve.



9. A function f has domain $[-5, 5]$ and a portion of its graph is shown.



   (a) Complete the graph of f if it is known that f is even.
   (b) Complete the graph of f if it is known that f is odd.

10. The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May it cost her \$380 to drive 480 mi and in June it cost her \$460 to drive 800 mi.

    (a) Express the monthly cost C as a function of the distance driven d, assuming that a linear relationship gives a suitable model.

    (b) Use part (a) to predict the cost of driving 1500 miles per month.

    (c) Draw the graph of the linear function. What does the slope represent?

    (d) What does C-intercept represent?

    (e) Why does a linear function give a suitable model in this situation?

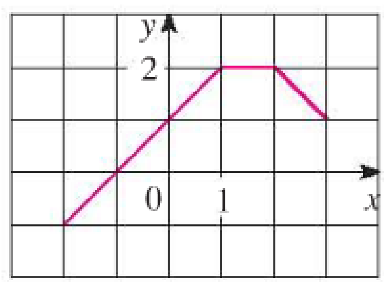
11. The graph of f is given. Draw the graphs of the following functions.

    (a) $y = f(x) - 2$

    (b) $y = f(x - 2)$

    (c) $y = -2f(x)$

    (d) $y = f(\frac{1}{3}x) + 1$



12. A spherical balloon is being inflated and the radius of the balloon is increasing at a rate of 2 cm/s.

    (a) Express the radius r of the balloon as a function of the time t (in seconds).

    (b) If V is the volume of the balloon as a function of the radius, find $V \circ r$ and interpret it.

13. If $f(x) = x + 4$ and $h(x) = 4x - 1$, find a function g such that $g \circ f = h$.

Optional Challenge Problem

Find a formula for a function $f(x)$ such that

- $f(3) = 0$
- $f(x)$ is even
- f has a horizontal asymptote at $y = 2$
- f has vertical asymptotes at $x = 4$ and $x = -4$
- $f(0) = 1$ (Meeting this requirement is the trickiest part!)