

Project 3: Infinite Series

Write up proofs for the following exercises in Abbott:

- (1) 2.7.13 (Abel's Test)
- (2) 2.7.14 (Dirichlet's Test)

Also write up proofs of the following **theorems**:

- (3) For any conditionally convergent series $\sum_{n=1}^{\infty} a_n$ and any real number $a \in \mathbb{R}$, there exists a rearrangement $\sum_{n=1}^{\infty} a_{\sigma(n)}$ converging to a . Here, $\sigma : \mathbb{N} \rightarrow \mathbb{N}$ is a bijection, also called a permutation.
- (4) The **alternating harmonic series** converges to $\ln 2$,

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln 2$$

(Note: we already know from the Alternating Series Test that the alternating harmonic series is convergent. The question here is, to prove that its limit is $\ln 2$.)