

MATH 2400: Calculus III

August 8, 2008

Exam 4

I have neither given nor received aid on this exam.

Name: _____

Box your answers.

In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show all of your work, and give adequate explanations.

No calculators, no books, no notes are allowed on this exam.

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	20 pts	
2	20 pts	
3	20 pts	
4	20 pts	
5	20 pts	
6	20 pts	
TOTAL	100 pts	

Unless otherwise stated, all curves are oriented counterclockwise when viewed from above, and surfaces are oriented upward/outward.

1. Evaluate the following integrals:

(a) $\int_C x^2 dx + xy dy + z^2 dz, \quad \mathbf{r}(t) = \langle \sin t, \cos t, t^2 \rangle, \quad 0 \leq t \leq \pi.$

(b) $\iint_{\sigma} xyz dS$ where σ is the portion of the unit sphere above the cone $z = \sqrt{x^2 + y^2}$.

2. Evaluate the following integrals:

- (a) $\int_C (2xy - \sin x) dx + (x^2 + 2y) dy$ where C is the line segment from $(0,0)$ to $(-5,7)$, followed by the arc of the parabola $y = x^2 + 6x + 12$ from $x = -5$ to $x = -3$, followed by the arc of the semicircle $x = \sqrt{18 - y^2}$ from $y = 3$ to $y = \sqrt{18}$.

- (b) $\oint \langle x, y, x^2 + y^2 \rangle \cdot d\mathbf{r}$ where C is the boundary of the portion of the paraboloid $z = 1 - x^2 - y^2$ in the first octant.

3. Evaluate the following integrals:

(a) $\int_C (4 + e^{\sqrt{x}}) dx + (\sin y + 3x^2) dy$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ in the first quadrant.

(b) $\iint_{\sigma} (\text{curl} \langle x^2 e^{yz}, y^2 e^{xz}, z^2 e^{xy} \rangle \cdot \mathbf{n} dS$, σ is the top half of the hemisphere $x^2 + y^2 + z^2 = a^2$.

4. In the following problems, find the flux of the vector field \mathbf{F} through the surface σ :

(a) $\mathbf{F} = \langle x^2, y^2, z^2 \rangle$, σ is the surface of the cube bounded by $x = 0$, $y = 0$, $z = 0$, $x = a$, $y = a$, $z = a$, ($a > 0$).

(b) $\mathbf{F} = \langle xze^y, -xze^y, z \rangle$, σ is the portion of $x + y + z = 1$ in the first octant, oriented downward.

5. TRUE or FALSE? IF true, justify your answer. Otherwise, give a counterexample or a complete explanation why it is false.

(a) If $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ for a simple closed curve C , then \mathbf{F} is conservative.

TRUE FALSE (circle one)

(b) For every closed surface σ , and vector field \mathbf{F} with continuous second partial derivatives, we have that $\iint_{\sigma} (\text{curl} \mathbf{F}) \cdot \mathbf{n} \, dS = 0$.

TRUE FALSE (circle one)

(c) If $\nabla^2 f (= \nabla \cdot \nabla f) = 0$ then $\int_C f_y dx - f_x dy$ is path independent in any simply-connected region.

TRUE FALSE (circle one)

(d) There is a vector field \mathbf{F} such that $\text{curl} \mathbf{F} = \langle 2x, 3yz, -xz^2 \rangle$.

TRUE FALSE (circle one)