Calculus 3 – Spring 2012
Homework 6
Due Friday 3/2/12

Problem 1. Let $f(\vec{v}) = \frac{1}{\|\vec{v}\|}$, where $\vec{v} = (x, y, z)$ and $\vec{v} \neq \vec{0}$. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$.

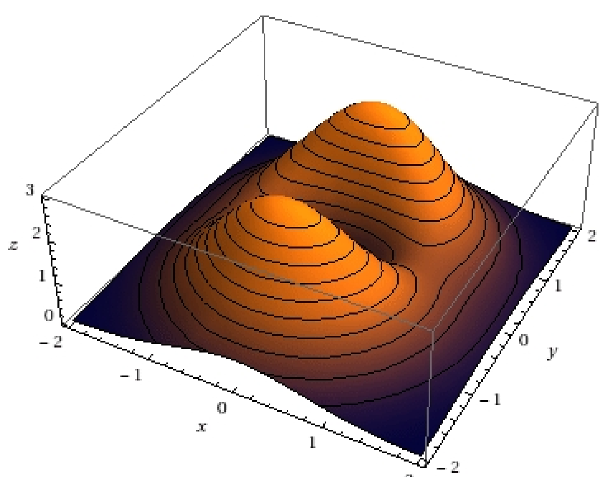
Problem 2. Show that the function

$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

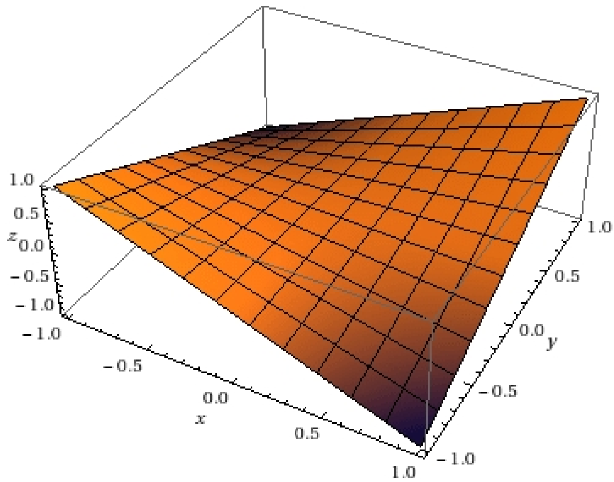
is not differentiable at $(0, 0)$. However, if $\gamma(t) = (x(t), y(t))$ is any differentiable curve passing through $(0, 0)$, then $f \circ \gamma$, that is $f(\gamma(t)) = f(x(t), y(t))$, is differentiable.

[Hint: You may suppose $\gamma(0) = (0, 0)$. Note that $f(\gamma(t))$ is a real valued function of a single variable, so just compute the single-variable limit definition of the derivative. Beware, however, that we may have $x'(0) = y'(0) = 0$. Use the fact that for any nonzero numbers a and b the following inequality holds, $\frac{a^3}{a^2+b^2} = a \cdot \frac{a^2}{a^2+b^2} \leq a \cdot \frac{\cancel{a^2}}{\cancel{a^2}} = a$.]

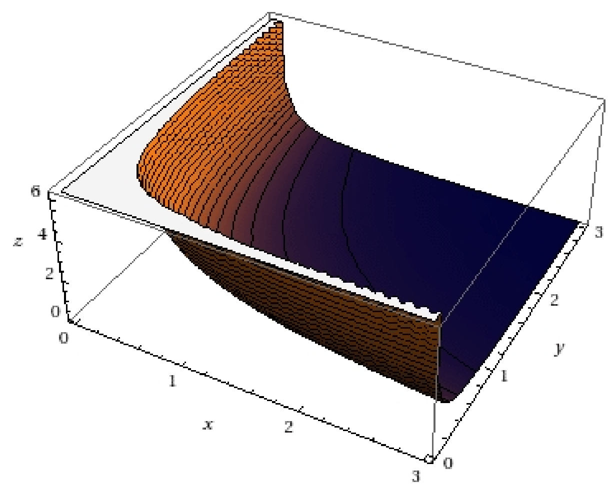
Problem 3. Find all the critical points of the function $f(x, y) = (x^2 + 3y^2)e^{1-x^2-y^2}$ and determine whether they are local maxima, local minima, or saddle points. [Note: The graph of f would suggest there are two maxima. Your job is to *prove* this using calculus, and moreover find the actual *coordinates* of those points.]



Problem 4. Find the absolute maximum and minimum values of the function $f(x, y) = xy$ on the rectangle $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. [The graph gives an easy way to check your answer, but you must *demonstrate* this using calculus. Show all your work.]



Problem 5. For $x > 0$ and $y > 0$, find the point on the graph of $f(x, y) = \dfrac{1}{xy}$ closest to the origin.



Problem 6. Maximize the function $f(x, y, z) = x + z$ subject to the constraint $x^2 + y^2 + z^2 = 1$.