Sections 14.4 and 14.5

1. In this problem, we re-derive a couple of formulas in a different way, using the gradient. First, consider the line in the xy-plane given by $y = mx + b$.

   (a) Give an example of a function $f(x, y)$ whose height 0 level curve (contour) has the equation $y = mx + b$.

   (b) Compute the gradient of $f(x, y)$.

   (c) Using the gradient, find the slope of a line perpendicular to the line $y = mx + b$. Then, write the equation of a line in the xy-plane with y-intercept b, perpendicular to $y = mx+b$.

   Now, let's do a similar calculation to find a formula for the normal vector of a plane.

   (a) Give an example of a function $f(x, y, z)$ whose 0 level curve is given by the plane $z = mx + ny + c$.

   (b) Use properties of the gradient of f to derive a formula for the normal vector to the plane $z = mx + ny + c$. Explain your solution carefully.

   (c) See, math is fun!

2. You are totally shredding on your skis, and you want to figure out what direction to go down the mountain in order to go the fastest (gnarly!). The shape of the mountain is given by a function $f(x, y)$. You have outfitted your board with a robot that computes the magnitude of the gradient, and the directional derivative in the direction the skis are pointing. Unfortunately, the robot is unable to compute the gradient, so you have to be clever to figure out the actual direction of steepest descent (ie. the opposite direction of the gradient vector). In the following three situations, use the information the robot gives you to determine how many degrees you should turn in order to go the fastest down the mountain. NOTE: The letters are in capitals, because it's a robot talking, obviously.

   (a) MAGNITUDE OF GRADIENT: 10. DIRECTIONAL DERIVATIVE IN DIRECTION YOU ARE POINTING: $-10\frac{\sqrt{2}}{2}$.

   (b) MAGNITUDE OF GRADIENT: 2. DIRECTIONAL DERIVATIVE IN DIRECTION YOU ARE POINTING: $-\sqrt{3}$

   (c) MAGNITUDE OF GRADIENT: 15. DIRECTIONAL DERIVATIVE IN DIRECTION YOU ARE POINTING: -11.5

3. At what point on the surface $z = 1 + x^2 + y^2$ is its tangent plane parallel to the following planes?

   (a) $z = 5$

   (b) $z = 5 + 6x - 10 - y$

4. Let $\overline{r}$ be the position vector of the point (x, y, z). If $\overline{\mu} = \mu_1\overline{i} + \mu_2\overline{j} + \mu_3\overline{k}$ is a constant vector, show that

$$grad(\overline{\mu} \cdot \overline{r}) = \overline{\mu}.$$

   Explain why this answer makes sense geometrically (hint: think of what $\overline{\mu} \cdot \overline{r}$ represents geometrically).