Calculus 3 – Spring 2012
Homework 11
Due Friday 4/20/12

Problem 1. Consider the parametrized plane $\mathbf{r}(s,t) = \langle 6,0,0\rangle + s\langle -2,1,0\rangle + t\langle 2,0,1\rangle$.

1. Find the equation of the plane in terms of x, y and z.
2. Find the equations of the two spheres of radius 3 whose tangent plane at the point $(2,4,2)$ is exactly the plane you found in part (a).
3. Parametrize the line passing through the centers of these two spheres.

Problem 2. Consider the sphere $x^2 + y^2 + z^2 = 25$. Use spherical coordinates to obtain a parametrization $\mathbf{r}(\phi,\theta) = \langle r_1(\phi,\theta), r_2(\phi,\theta), r_3(\phi,\theta)\rangle$, and compute $\mathbf{r}_\phi \times \mathbf{r}_\theta$ at the point $(3,4,0)$ (here, $\mathbf{r}_\phi = \langle r_{1\phi}, r_{2\phi}, r_{3\phi}\rangle$ and $\mathbf{r}_\theta = \langle r_{1\theta}, r_{2\theta}, r_{3\theta}\rangle$). Denoting this vector by $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$, show that $n_1(x-3) + n_2(x-4) + n_3(z-0) = 0$ is precisely the tangent plane to the sphere at $(3,4,0)$ obtained by the methods of chapter 14.

Problem 3. Calculate the line integral of $\mathbf{F} = x\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ along the line between the points $(0,-2,0)$ and $(0,-10,0)$.

Problem 4. Give conditions for constants a and b to ensure that the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is positive if $\mathbf{F} = ay\mathbf{i} - ax\mathbf{j} + (b-1)\mathbf{k}$ and C is the line segment from the origin to $(1,1,1)$.

Problem 5. Let $C_1(t) = (2t, 2t)$, where $0 \le t \le 1/2$, and $C_2(t) = \left(\frac{t^2-1}{3}, \frac{t^2-1}{3}\right)$, where $1 \le t \le 2$, be two parametrizations of the line segment ℓ from $(0,0)$ to $(1,1)$. Calculate the line integral of $\mathbf{F} = (3x-y)\mathbf{i} + x\mathbf{j}$ along ℓ using each of the two parametrizations C_1 and C_2.