Calculus I Final Review

1. Find an equation for an exponential function through the points $(1,5)$, $(3,12)$ and with horizontal asymptote $y=0$. Find another equation when the asymptote is $y=20$.

2. Suppose the half-life of a radioactive isotope is 125 days. How many days will it take for a sample of the substance to reach 10% of its starting amount.

3. A sinusoid has a minimum at $(32,-2)$ and a maximum at $(38,8)$ with no critical points in between. Find two equations for this function, one in terms of sin and the other in terms of cos .

4. Let $f(x) = \begin{cases} x^3 - 2x^2 + 3x - 2 & \text{if } x \le c \\ 2x - 2 & \text{if } x > c \end{cases}$ for some constant c. For what value(s) of c is f continuous? Differentiable?

5. Evaluate the following:

   (a) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{8x^2+1}$

   (b) $\displaystyle\lim_{x\to 0^+} (\sin x)^{\ln x}$

   (c) $\displaystyle\lim_{x\to 1} (\ln x)^{\ln x}$

   (d) $\displaystyle\lim_{x\to \ln 2} \frac{\sinh x - \frac{3}{4}}{x - \ln 2}$

   (e) $\displaystyle\lim_{x\to\infty} x - \sqrt{x^2+x}$

   (f) $\displaystyle\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$

   (g) $\displaystyle\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$

6.

t (sec)	0	1	2	3	4	5
$v(t)$ (ft/sec)	40	32	26	22	20	19

   (a) Approximate the instantaneous acceleration $t=2$ sec.

   (b) Find the average acceleration over the first 4 seconds.

   (c) Find an upper/lower bound for the total distance traveled over the 5 seconds.

   (d) Suppose $s(0) = 15$ ft. Approximate $s(t)$ at each second.

7. Using the limit definition, find the derivative of $f(x) = \frac{1}{x^2+1}$.

8. Let $V(t)$ be the volume of a growing yam, where t is measured in days and $V(t)$ in cm^3. Interpret the following, with units:

   (a) $V(40) = 22$

   (b) $V'(35) = .8$

   (c) $V^{-1}(25) = 60$

   (d) $(V^{-1})'(22) = 1.25$

   (e) $\displaystyle\int_{10}^{20} V'(t)\,dt = 11$

   (f) Also, using the above, evaluate $V'(40)$.

9. Suppose f is a decreasing, continuous, concave down function with $f(0) = 6$ and $f'(0) = -2$. How many zeroes can f have and where can they occur? Why? Also, can $f(-2) = 12$? $f(-2) = 4$? $f(-2) = 8$? $f(-2) = 10$?

10. Find the tangent line to $xy^2 + y^3 = x^2 + 8$ at $(0, 2)$.

11. Verify that $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.

12. Approximate $\ln(1.02)$ without a calculator. Is this an over or under approximation?

13. Let $f(x) = 3x^4 - 4x^3 + 6$. On what intervals is f increasing/decreasing? Concave up/down? Find all critical points and any local max/mins. Find the global max/mins on [-2,2].

14. Find the point on the parabola $y = x^2$ that is closest to $\left(2, \frac{1}{2}\right)$.

15. A cylinder is changing size, but is keeping the same volume, $100\pi\,\text{cm}^3$. At some point, the height is 25 cm and the radius is decreasing at a rate of 3 cm/min. At what rate is the height changing at this moment?

16. Consider the graph defined by $(x, y) = (e^t, 5e^{2t})$.

    (a) Find the parametric equations for the tangent line at $t = \ln 3$.

    (b) Find the equation of the tangent line expressing y as a function of x.

    (c) Find the speed at $t = \ln 3$.

    (d) Is this graph the same as $(x, y) = (t, 5t^2)$? as $(x, y) = (t^2, 5t^4)$? as $(x, y) = \left(\frac{1}{t}, \frac{5}{t^2}\right), t > 0$?

17. Find the area between $y = x^2$ and $y = x^3$.

18. Suppose f is even, $\displaystyle\int_2^3 f(x)\,dx = 5$, and $\displaystyle\int_{-3}^0 f(x)\,dx = 2$. Evaluate $\displaystyle\int_0^5 f(x-3) - 2\,dx$.

19. How does $\left|\displaystyle\int_a^b f(x)\,dx\right|$ compare to $\displaystyle\int_a^b |f(x)|\,dx$?

20. Find a number c such that the average value of $f(x) = \dfrac{1}{x}$ on $[c, 2c]$ is 1.

21. A rock on another planet falls from a height of 100 m and hits the ground after 5 seconds. What is the acceleration due to gravity on the planet?

22. Solve the initial value problem $\dfrac{dy}{dx} = e^x + 4\sin x$, $y(0) = 2$.

23. Define $F(x) = \displaystyle\int_2^{x^2} \frac{1}{t^2+1}\,dt$. Find $F(\sqrt{2})$, $F'(\sqrt{2})$, and $F''(\sqrt{2})$.